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MEASUREMENT OF THE POINTING ACCURACY OF A SATELLITE GROUND TERMINAL ANTENNA BY USING RADIO STARS, SUN, AND MOON

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Communications Division

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SUMMARY

Satellite ground terminals require accurate read-out devices for the elevation and azimuth angles of the antenna beam centre. For the calibration of these devices the known positions of stellar radio sources can be used. Usually, the measurements are carried out when the radio source is moving in elevation only or in azimuth only. In this memorandum a method is developed for performing the measurements at any point of the trajectory and for then determining the antenna misalignments by using a best-fit procedure based on a large number of measurements. The particular sources considered are the radio stars, the sun, and the moon.

The accuracy of the method is analyzed by estimating the errors associated with the various parameters involved in the measurements, and a way of deriving the accuracy from the scatter of the measured results is described. A calibration accuracy of 0.006° is found to be feasible.

The method has been applied to the experimental ground terminal SET-2 at the SHAPE Technical Centre. Within the accuracy limits of 0.01° for each axis no misalignments could be detected. The error derived analytically agrees satisfactorily with the error derived from the variance of the observations.

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1. INTRODUCTION

A satellite ground terminal antenna which is steerable and highly directive requires accurate read-out devices for the elevation and azimuth angles of the radio beam axis. The calibration accuracy should be in the order of 0.01° for quick acquisition of the satellites, collection of tracking data for orbit determination, programme tracking, and radio-astronomical observations. Inaccurate antenna pointing may be due to non-coaxial fitting of the angle encoders on the antenna axes, distortion of the dish when moved in elevation, and imperfect levelling of the azimuth bearing.

The pointing accuracy of a satellite ground terminal can be measured by means of landmarks, a satellite, or stellar radio sources. The advantages of the radio-source method are that the calibration points cover almost the entire range of elevation and azimuth, that radio sources are available at any convenient time, and that the positions of the radio sources are very accurately known.

In order to obtain the antenna misalignments in elevation and azimuth separately, the radio sources are usually measured at their turning points when they move in elevation only or azimuth only. Furthermore, because of the low flux of radio stars, a clear sky is necessary. Coincidence of these two conditions within the measurement periods dictated by the work programme of the ground terminal is rather rare. Hence, this method will in general provide insufficient data.

In this memorandum, therefore, the calibration method using radio stars, sun, and moon (Ref. 1, Chapter 4) is further developed to permit measurements at any (visible) point of the trajectory of the radio source. Results are derived by best-fitting procedures from a large number of measurements, thereby randomizing the systematic errors and leading to improved measurement accuracy. For the data processing a computer program is given providing the misalignments in elevation and azimuth and the rms errors. For the computation of the celestial position of the radio sources a programmable desk calculator together with an astronomical handbook is used. For the convenience of the user, the appropriate formulae and constants have been extracted and summarized in the appendices to this memorandum.

A thorough error analysis assesses the feasibility of the various radio sources for antennas of different sizes. The error limits are evaluated analytically depending on whether a radio star, the sun, or the moon is used. The error limits are also derived from the spread of the measurement results.

The method has been applied to the calibration of the satellite ground terminal SET-2 of the SHAPE Technical Centre. The feasibility of the method is demonstrated and the results are reported together with the associated errors.

2. LOOK ANGLES OF THE RADIO SOURCES

2.1 GENERAL

The apparent elevation and azimuth angles of a radio source in the sky as seen from the ground terminal - in short, the look angles - may be calculated in two ways. One approach is to program on a computer the equations of motions of the celestial objects. This is relatively simple for fixed stars, it becomes more difficult for the sun, and is very involved for the moon. The look angles are then available in the form of voluminous tables.

In this memorandum a different approach is followed. The position of the star, sun, or moon for the intended time of the measurement is taken from the handbook "The Astronomical Ephemeris" (Ref. 2, below designated AE) of the current year in terms of the inertial coordinates of the star and these are transformed by means of relatively easy calculations into the look angles for the particular ground terminal. For these computations a programmable desk calculator, such as the HP 9100B, has proved eminently suitable for getting results of the required accuracy within a few seconds. Thus advantage can be taken of a sudden clear sky period by calculating the look angles in "real time" immediately before the measurements.

2.2 COORDINATES OF THE GROUND TERMINAL

For the calculation of the look angles the location of the ground terminal on the earth is required. Usually the geodetic coordinates, longitude and latitude, are given, based on the reference ellipsoid 1950 European Datum. Figure 1 shows the ground terminal relative to the reference ellipsoid and the definition of the geodetic latitude ϕ by means of the normal to the ellipsoid, which is called the vertical. The geodetic longitude λ is defined as the angle measured on the equator between the meridian of the ground terminal and the Greenwich meridian, and reckoned positive to the east.

Mass anomalies within the earth deflect a plumbline slightly from the normal to the reference ellipsoid: this effect may be as much as 0.02°. The plumbline defines the astronomical latitude ϕ_a (Fig. 1) and longitude λ_a. The deflection of the plumbline can be resolved into the two components ξ_p and η_p which are the deflections to the north and east respectively. The astronomical coordinates ϕ_a and λ_a are related (Ref. 3, Chapter 5) to the geodetic coordinates ϕ and λ by

$$\phi_a = \phi + \xi \qquad (1)$$

$$\lambda_a = \lambda + \eta/\cos\phi \qquad (2)$$

The azimuth bearing of the ground terminal is usually adjusted to be horizontal by means of a spirit level or pendulum. Therefore the elevation and azimuth angles displayed at the ground terminal refer to the local plumbline and the astronomical latitude and longitude have to be used. The astronomical latitude and longitude for SET-2 are given in Ref. 4.

Because of the large distance of fixed stars the displacement of the ground terminal from the centre of the earth can be neglected in all calculations. For the sun, however, and even more essentially for the moon, the geocentric distance ρ of the ground terminal and its geocentric latitude ψ as shown in Fig. 1 are taken into account. However, the height of the ground terminal above the ellipsoid is neglected. The equations given in Ref. 3, Chapter 2 for ρ and ψ are simplified by means of the approximation $f \ll 1$ ($f=(a-b)/a=$ flattening $= 1/297$) yielding

$$\psi = \phi - \frac{1}{297} \sin 2\phi \qquad \text{(radians)} \qquad (3)$$

$$\rho = a(1 - \frac{1}{297} \sin^2 2\phi) \qquad (4)$$

on the reference ellipsoid 1950 European Datum.

2.3 CORRECTION FOR THE TILT OF THE AZIMUTH PLANE

Although means are usually provided to level the azimuth bearing, in practice a residual deflection of the normal to the azimuth plane from the plumbline is observed. This tilt can be measured accurately (Ref. 4).

The inaccuracy of the look-angles due to the azimuth plane tilt could be considered as part of the overall error to be evaluated. However, it is helpful for finding other sources of error to separate this effect and to apply the calibration to the read-out devices after a correction for azimuth plane tilt has been made. The deflection of the normal to the azimuth plane is treated similarly to the deflection of the plumbline by introducing a corrected latitude ϕ_t and longitude λ_t given by

$$\phi_t = \phi_a + \xi_t \tag{5}$$

$$\lambda_t = \lambda_t + \eta_t/\cos\phi \tag{6}$$

where

ξ_t = deflection of the normal to the north

η_t = deflection of the normal to the east

The numerical values found (Ref. 4) for the ground terminal SET-2 are

$$\phi_t = 52.08517^\circ$$

$$\lambda_t = 4.31863^\circ E$$

2.4 LOOK ANGLES OF FIXED STARS

The position of a star on the celestial sphere is defined by two angles, namely the right ascension and the declination. The right ascension is the angle between the First Point of Aries and the meridian of the star measured in the equatorial plane positive in the direction opposite to the apparent rotation of the celestial sphere. The declination δ is the angle between the star and the equator measured in the meridianal plane, positive to the north, see Fig. 2.

Stars with a declination

$$\delta > 90^\circ - \phi \tag{7}$$

are visible at all times of the day: their trajectory as seen from an observer on the rotating earth is an ellipse. Their upper culmination is in the north if $\delta > \phi$ (example Cassiopeia), their upper culmination is in the south and $\delta < \phi$ (example Cygnus). The lower culmination is always in the north.

Stars with a declination where

$$\phi - 90^\circ < \delta < 90^\circ - \phi \tag{8}$$

set below the horizon for part of the day and have their culmination in the south (example Taurus).

Figure 3 plots elevation against azimuth for these three stars (and also the sun and the moon) as seen from SET-2.

Stars with a declination of

$$\delta < \phi - 90^\circ \tag{9}$$

are never visible at the observer's latitude.

The three strongest radio stars suitable for measurements at X-band frequencies are Cassiopeia A, Taurus A and Cygnus A. Their right ascensions and declinations are given in Appendix A.

Although for the purpose of the diagram in Fig. 3 the positions of the stars can be considered as fixed, for a precise calculation of their look angles a secular change of right ascension and declination due to the precession of the earth of the order of 0.01° per year has to be taken into account. The appropriate constants are published annually in the AE.

For the convenience of the user, all relevant formulae from Ref. 5 and the necessary constants of the stars and the corrections for the precession from 1972 to 1975 are collected in Appendix A. However, the current AE must be consulted to find the hour angle of the First Point of Aries at 0^h UT for the day of the measurement. The elevation and azimuth angles can be most easily calculated by the HP 9100B desk calculator program also given in Appendix A. Once the parameters for the day of the measurement have been entered the look angles can be displayed for any instant of this day by only keying in the time.

2.5 LOOK ANGLES OF THE SUN

In principle, look angles of the sun are calculated in the same way as for the fixed stars. However, the sun relative to the celestial sphere goes through an inclined circle (ecliptic) once a year. Therefore over the year the right ascension varies from 0 to 360^o, and the declination from + $23,5^o$ to -23.5^o. Right ascension and declination for the sun are tabulated for every day in the AE and values can be linearly interpolated in between.

The distance of the ground terminal from the centre of the earth is no longer negligible compared to the distance of the sun. The apparent displacement of the sun if it could be observed from the centre of the earth is the parallax. The distance of the sun, and hence the parallax, varies over the year. In the context of this memorandum this second order effect is neglected.

All formulae required are collected in Appendix B. An HP 9100B desk calculator program is also given. This, used with the AE of the current year, enables the look angles of the centre of the sun to be calculated, once the appropriate constants for the day of the measurement have been entered.

2.6 LOOK ANGLES OF THE MOON

The right ascension and declination of the moon vary considerably with time because the moon drifts through a full circle on the celestial sphere once a month. The orbital plane of the moon is inclined to the ecliptic at 5° and the intersecting line of these planes rotates, completing one revolution in 18.6 years. Hence, the declination of the moon varies by 57° and the azimuth by 360° over a month. The declination may be as high as 28.5° and as low as -28.5°, depending on the year. In any case the moon culminates in the south for European observers and part of the trajectory is below the horizon. Declination and right ascension of the moon are tabulated in the AE for every hour. Times in between can be linearly interpolated.

The distance of the ground terminal from the earth's centre is not negligible compared to the distance of the moon, and hence a parallax correction is necessary. Furthermore, the distance of the moon and hence the parallax correction of the observer vary during the month. The horizontal parallax of the moon is tabulated hourly in the AE and is used for the calculation of the look angles.

All formulae required are collected in Appendix C. An HP 9100B desk calculator program is also given. This, together with the AE of the current year, enables the calculation of the look angles of the centre of the moon for a period of one hour. For the next hour new data from the AE have to be entered.

2.7 CORRECTION FOR THE REFRACTION

Due to the refraction in the earth's atmosphere a radio beam coming from space is bent towards the ground. Therefore, the elevation of a radio source as seen from the ground terminal is higher than indicated in Sections2.4 to 2.6. The azimuth angle is not affected because an atmosphere undisturbed in the horizontal direction is assumed. The correction for the refraction depends on the surface refractivity of the dry air, N_d, and on the refractivity of water vapour, N_w. Both vary geographically and seasonally. However, in order to arrive at mean values, we assume an earth surface temperature of 15°C, an atmospheric pressure of 1000 mb, and a water vapour pressure of 12 mb. Inserting these values into the formulae recommended by the CCIR (Ref. 6) we get

$$N_d = 269$$

$$N_w = 54$$

In Ref. 7 formulae for the elevation correction ΔE are given as a function of elevation angle E and the refractivities N_d and N_w. Inserting the above numerical values for N_d and N_w gives ΔE as a function of E, and this is plotted in Fig. 4.

After the elevation angle has been calculated as indicated in Sections 2.4 to 2.6 the correction ΔE is obtained from Fig. 4 and added to the elevation to give

$$E_{corr} = E + \Delta E . \tag{10}$$

3. MEASUREMENT OF THE ANTENNA POINTING ACCURACY

3.1 MEASUREMENT PRINCIPLE

The look angles of a radio source at a required time instant are calculated as outlined in Chapter 2. A few minutes before that instant the antenna is set to these calculated look angles by using the antenna's angle read-out devices. While the radio star drifts through the beam pattern of the antenna the received power is recorded. A peak will be noted when the star goes through the beam centre. The time instant of the peak can be determined by means of time marks which are recorded simultaneously. If the antenna were pointing exactly in the true direction of the star, the peak would occur exactly at the predicted instant. However, as it is most likely that the antenna will have an offset in elevation and azimuth in respect to the read-out devices, the peak power will be received earlier or later than calculated. The method proposed in this memorandum enables the offsets in elevation and azimuth to be determined from a number of measurements of this time difference.

Figure 5 shows the relationships between the parameters involved in the measurement. At the time instant t_1, the star is in the position P_1 with elevation E_1 and azimuth A_1. The antenna is intended to point at P_1 but because of the misalignments ΔA in azimuth and ΔE in elevation the antenna in fact points at P_A. The antenna beam pattern is assumed to be circular; a number of lines of constant antenna gain are shown. The star moves along its trajectory P_1P_2. At the receiver, maximum power will be recorded when the star trajectory is closest to the antenna beam centre. At this instant, corresponding to point P_2, the trajectory is tangential to a circle of constant antenna gain. Therefore, the angle between the lines P_2P_1 and P_2P_A is a right angle. Having so determined the instant t_2, the look angles of the star at t_2, E_2 and A_2, can be calculated by using the formulae of Chapter 2.

Rather than proceeding with the cumbersome equations for spherical triangles we recall that the lengths d and r (Fig. 5) are small angles (<0.1°) which allows us to treat the triangle $P_1P_2P_A$ as the plane triangle shown in Fig. 6. The unknowns ΔE and ΔA are related to the known parameters E_1, A_1, E_2, A_2 by

$$d = r \cos (\gamma - \beta) \tag{11}$$

where

$$d = \sqrt{\{(E_2-E_1)^2 + (A_2 - A_1)^2 \cos^2 E_1\}} \tag{12}$$

$$\beta = \arctan \frac{(A_2 - A_1)\cos E_1}{E_2 - E_1} \tag{13}$$

$$r = \sqrt{\{\Delta E^2 + (\Delta A \cos E_1)^2\}} \tag{14}$$

$$\gamma = \arctan \frac{\Delta A \cos E_1}{\Delta E} \tag{15}$$

As there are two unknowns ΔE and ΔA (or r and γ by means of (14) and (15)) two equations (11) are required; this calls for two measurements with different trajectory angles β. However, because of inevitable errors in measuring E_1, E_2, A_2, A_1 (or d and β by means of (12) and (13)) a much better accuracy can be achieved by making a large number of measurements and finding ΔE and ΔA by the method of least squares.

Let E_{1i}, E_{2i}, A_{1i}, A_{2i} (i = 1,...,n) be found as the result of n measurements. Then by using (12) and (13) the corresponding quantities d_i and β_i can be obtained. The presence of errors in measuring the times of the peak power means that the measured d_i differs from the true d_i by small deviations v_i; hence (11) leads to a set of n equations

$$d_i + v_i = r_i \cos (\gamma_i - \beta_i) \quad i = 1,\ldots,n . \tag{16}$$

The "best fitting" values of the unknowns ΔE and ΔA are found by varying them in such a way that the sum of the squared errors v_i becomes a minimum. Thus we have

$$F(\Delta E, \Delta A) = \sum_{i=1}^{n} v_i^{\,2} = \sum_{i=1}^{n} \left[r_i \cos(\gamma_i - \beta_i) - d_i \right]^2 = \text{Min!} \quad (17)$$

To illustrate this process, the function F (ΔE, ΔA) resulting from 36 measurements at SET-2 is plotted 3-dimensionally in Fig. 7. It can be seen that there is an unambiguous minimum at about $\Delta E = 0$, $\Delta A = 0.01^{\circ}$ surrounded by quite steep slopes.

For the evaluation of a set of equations (16) a computer program called ANTOFF has been written incorporating a standard subroutine for finding the minimum at (17). It is given in Appendix D.

3.2 MEASUREMENT TECHNIQUE

Essentially the measurement consists in recording the received power of the radio source and time marks. The radio sources differ considerably with respect to their fluxes. As their radiation is noise of constant spectral density over the measurement bandwidth, the received power may be expressed by an equivalent noise temperature given by

$$T = \frac{G\ S\ \lambda^2}{4\ \pi\ k} \quad (18)$$

where G = antenna gain, S = flux, k = Boltzmann constant, and λ = wavelength. If we take the flux densities and the correction for the angular dimension of the sources as given in Ref. 8 the equivalent noise temperatures of the sources are as shown in Table 1.

Table 1

Equivalent noise temperatures of radio sources at 7 GHz in $^{\circ}$K

Source	Antenna diameter*			
	60'	40'	20'	10'
Sun	~12,000	~22,000	~15,000	~6,700
Moon	2,400	2,100	1,400	620
Cassiopeia A	71	35	10	3
Taurus A	69	33	9	2
Cygnus A	28	13	3	1

***** *70% efficiency assumed and Gaussian shape of the source's radiation and the antenna pattern.*

From this table it can be seen that the power radiated by the sun and the moon is sufficiently high to be recorded by an ordinary linear receiver even for ground terminal antennas of only 10-feet diameter.

The noise power radiated by radio stars, however, is several orders of magnitude smaller. For a receiver with an uncooled parametric amplifier, the system noise temperature of the ground terminal receiver is about 250°K. The stability of the receiver gain over half an hour is about 0.05 dB corresponding to an output noise power variation equivalent to 3°K. Hence to distinguish the received power of the radio star from system noise variations the radio star temperature should be much greater than 3°K, say at least 30°K. This condition is fulfilled by ground terminals with antenna of 40 feet or more diameter and for the radio sources Cassiopeia and Taurus.

Ground terminals with smaller antenna diameters receiving radio stars need a radiometer which eliminates the effect of gain drift. This is achieved by continuously switching the receiver input between the antenna and a reference noise source and at the output synchronously detecting and displaying the difference voltage. As changes in receiver gain and receiver noise figure are common to both channels, to a first order they are cancelled at the output.

In order to smooth the post-detector noise voltage, the output signal usually passes an RC filter with time constant τ. When a radio source passes through the antenna beam pattern, due to the radiometer time constant the occurrence of the peak is delayed. It has been shown (Ref. 9) that the peak is shifted by τ provided that τ does not exceed one-tenth of the half-power transit time of the star. Hence, in determining the instant of the peak from the recording, the time τ must be subtracted. The half-power transit time can be determined from the angular velocity w of the star, which is (Ref. 1, Chapter 5)

$$w = 0.25 \cos \delta \text{ (degrees/minute)}$$

where δ = declination,

and the 3-dB beam-width of the antenna. The fastest star would be one with $\delta = 0$ travelling at an angular velocity of 0.25°/min. For example, for a beamwidth of 0.26° (SET-2) the fastest transit time is one minute. Therefore, the radiometer time constant must not exceed six seconds in order to maintain the validity of simply subtracting τ as correction.

The post-detection noise sets a limit below which radio stars cannot be recorded. The post-detection noise power is equivalent to a noise temperature ΔT given (Ref. 1, Chapter 3) by

$$\Delta T = \frac{T_s}{\sqrt{(B\tau)}} \qquad (20)$$

where

T_s = system noise temperature

B = RF bandwidth

τ = radiometer constant

As an example let $T_s = 250^oK$, $B = 20$ MHz, and $\tau = 5s$ then we have $\Delta T = 0.03^oK$. This noise should be small (-15 dB) compared to the signal. Hence, from Table 1 it can be seen that only for ground terminals with an antenna diameter of less than 20 feet is it not possible to track radio stars, the others are able to receive at least Cassiopeia and Taurus. The sky should be clear in any case because otherwise the noise temperature fluctuations of the atmosphere would mask the radio star signal.

Examples of radio source recordings are given in Figs. 8 and 9 which show respectively a sun recording and a recording of Cassiopeia A taken at SET 2.

3.3 DATA PROCESSING AND ANALYSIS

A measurement and its analysis consists of the following steps.

(1) Select radio source; calculate an apparent position E_1, A_1 by the formulae or desk calculator programs given in Appendices A to C (computational accuracy of 10^{-4} degrees required); add to the elevation angle the refraction correction ΔE from Fig. 4.

(2) Direct the antenna to this calculated position and record the received power as the radio source drifts through the antenna beam. Also record time (UTC) marks.

(3) Read the instant of peak received power; when a radiometer is used subtract its time constant τ.

(4) Calculate the apparent position E_2, A_2 of the radio source at the instant of the peak of the transit as outlined in Appendices A to C.

(5) Perform as many measurements as possible. Then run the computer program given in Appendix D whereby the input values are quadruples of the elevation and azimuth angles (E_1, A_1: E_2, A_2) obtained in (1) and (4).

In the calculations it is only the difference between the elevation angles E_1 (see step 1) and E_2 (see step 2) that is used, and so it is convenient to insert them both uncorrected into the computer program. The results of the computer run are the antenna misalignments, ΔE and ΔA, which best fit the measured values.

4. ERROR ANALYSIS

4.1 ERROR OF THE LOOK ANGLES OF THE RADIO SOURCE

In the measurement described in Chapter 3 there are two principal sources of error: first, the look angles to which the antenna is set at the start of the measurement may be determined with an error; and second, the occurrence time of the peak of the received power will be read with a certain error. This section deals with effects related to the first kind of errors, Section 4.2 with the second kind.

So far the radio sources have been treated as strict point sources of radiation. All of the radio sources, however, have a finite angular extension. Nevertheless the calculations hold for a circularly symmetric distribution of radiation and a circular antenna pattern. This condition is fulfilled for Cassiopeia; the other sources have a non-circular distribution of radiation.

The radio stars Taurus and Cygnus have an asymmetry of 0.025° (Ref. 8). The sun is neither circularly symmetric nor is its radiation constant with time. It has been observed (Ref. 10) that the effective centre of the radiation in the mean is displaced by 0.025° from the astronomical centre. The correlation time for the effective centre is five days; therefore, repeating the measurements at intervals of not less than five days permits this displacement to be treated as random.

The moon is a dependable radio source due to the periodical illumination. The radiation is a super-position of a periodically varying component with a constant component. At 7 GHz the brightness temperature of the moon varies between 207° and 219° lagging 3.3 days with respect to the optical illumination (Ref. 8). The effective centre of radiation is shifted to the brighter side of the moon. In Ref. 1, Chapter 4, an equation is given from which a maximum shift of 0.009° is calculated from the above values.

In practice, it is usual for all sources to be used in the calibration measurements. In order to arrive at a single value for the error due to the finite extension of the radio sources it is assumed that all five sources are measured an equal number of times. The average error σ_m is then the mean of the individual errors, so we have

$$\sigma_m = \frac{0 + 0.025 + 0.025 + 0.025 + 0.009}{5} = 0.017^{\circ}$$

In addition to this error in the position of the radio emission centre due to the finite extension of the source there are other causes of error such as:

- mathematical approximations
- error in the coordinates of the ground terminal
- error in the right ascension α and declination δ of the star ($\sigma_\alpha = 0.008^{\circ}$, $\sigma_\delta = 0.007^{\circ}$)
- short term irregularities of the motion of the earth
- aberration.

Each of these errors (except those in right ascension and declination) is less than 0.003° and thus small compared with the error described above. One can therefore account for all errors by attributing an rms error of 0.01° for the combination of all effects.

In calculating the look angles of the stars the elevation angles found mathematically are corrected for atmospheric refraction (Section 2.7). This correction introduces yet another error. The correction, and consequently its error, is large at elevation angles below 20°. The correction curve in Fig. 4 is based on a stratified model of the earth's atmosphere which neglects turbulences and uses only the refractivity at the earth's surface. In Ref. 11 it is shown that the error resulting from these approximations does not exceed 0.003°. In Section 2.7 a mean refractivity with respect to seasonal and geographical variations is used. On the charts for the refractivity given in Ref. 6, in the NATO area variations of the refractivity over the range N = 310 - 360 are found. This corresponds to a variation in the elevation correction of less than 0.02^{o} for elevations above 10^{o}.

By treating all errors as random a combined rms error for the position of the star can be given. From the above figures an overall error of 0.028^{o} results.

4.2 ERROR IN DETERMINING THE INSTANT OF THE TRANSIT

The error estimated in the previous section enters the measurement at the start. An additional error arises when the recording of the star transit is analyzed and the time instant of the peak is determined. The following sources of error are involved in the determination of the peak:

- receiving system noise
- asymmetry of the source's radiation pattern
- asymmetry of the antenna pattern
- correction for the time constant of the radiometer
- time marks.

The error due to system noise applies mainly to the radio stars because of their low flux density. For the sun and the moon this source of error is negligible as can be seen by comparing Figures 8 and 9. When the radiometer is used the mean post-detection power fluctuation, expressed in equivalent noise temperature ΔT, is given by (20). The power uncertainty can be related to an uncertainty in angle by means of transit curves. For a point source the transit curve is an image of the antenna pattern, hence (Ref. 12)

$$T = T_{peak} \left[\frac{2J_1(\sqrt{(G)}.\sin\theta)}{\sqrt{(G)}.\sin\theta} \right]^2 \approx T_{peak} \left[1 - \tfrac{1}{4} G\theta \right]^2 \qquad (21)$$

where θ = off-axis angle << 1 and a uniformly illuminated circular aperture is assumed. Inserting (19) and (20) into (21) yields the off-axis angle $\Delta\theta$ which corresponds to the post-detector noise temperature fluctuation ΔT: we obtain

$$\Delta\theta = \left[\frac{16\ \pi\ k}{\sqrt{(B\tau)}.S\lambda^2} \cdot \frac{T_s}{G^2}\right]^{\frac{1}{2}} \tag{22}$$

It can be seen that the resolution $\Delta\theta$ of the measurement improves linearly with increasing antenna gain G but only with the square root of decreasing system noise temperature T. For Cassiopeia at 7 GHz and assuming $T_s = 250^\circ K$, B = 20 MHz, antenna efficiency = 0.7, and τ = 1/10 of the transit time, Table 2 gives the attainable angle resolution for various antenna diameters.

Table 2

Angular uncertainty due to radiometer post-detection noise

Antenna diameter	60'	40'	30'	20'	10'
Angle resolution	0.0016°	0.004°	0.007°	0.013°	0.05°

For a very high gain antenna and consequently a high directivity the beamwidth is comparable to the angular extension of the radio sources. For the moon and the sun this is always true. In this case the recording is an image of the radiation distribution of the source rather than of the antenna pattern. The peak part of the recording flattens and makes the determination of the maximum more uncertain. This error has been already accounted for in Section 4.1, and it masks the error due to noise for high gain antennas (> 40').

The procedure for measuring the misalignment of the antenna is based on circular symmetry of the antenna pattern. Depending on the feed system, however, the beamwidth is a function of the orientation of the cross-section through the antenna pattern. No generally valid error can be given for this effect. Typically an error of 0.01° may be assumed.

The time instant of the peak of a radiometer recording is corrected for the radiometer time constant. This time constant can be measured with an accuracy of at least one second. Hence, the corresponding angle error is 0.004° at most.

The setting of the clock and the resolution of the time marks is within one second, leading to an error of not more than 0.004°.

By taking a large number of measurements and by using different radio sources, different points on their trajectories, and measurements on different days it can be assumed that all errors occur randomly and therefore can be added rms-wise. As an example we sum the errors discussed in this section for a 40' terminal measuring Cassiopeia, Taurus, sun, and moon an equal number of times:

Error due to noise:		
	Cas	0.004°
	Tau	0.004°
	Sun	negl.
	Moon	negl.
	mean:	0.002°
Antenna pattern asymmetry	:	0.010°
Error of radiometer time constant (note that half of the measurements do not involve the radiometer)	:	0.002°
Time marks	:	0.004°
	rms sum	0.011°

We now combine the rms error of the position of the effective emission centre of the radio source derived in Section 4.1 with the rms error due to the uncertainty in determining the instant of peak received power derived in this section. This yields a total error of

$$\sigma = \sqrt{(0.028^2 + 0.011^2)} = 0.03^\circ$$

which is the error for one single measurement. The antenna misalignment is derived from a number of independent measurements each having this error. As two measurements are the minimum necessary to solve for the two unknowns, n measurements have n - 2 degrees of freedom leading to the error of the mean

$$\sigma_m = \frac{\sigma}{\sqrt{(n - 2)}} \quad . \tag{23}$$

Thus, with 30 measurements, for example, we have

$$\sigma_m = \frac{0.03}{\sqrt{28}} = 0.006^\circ$$

4.3 ERROR DERIVED FROM THE SCATTER OF THE MEASURED RESULTS

If it can be assumed that by making a large number of measurements all pertinent parameters are varied randomly and systematic errors are eliminated then the overall error can be derived statistically from the scatter of the results. The parameters d_i in (16), which are the measured angles between the indicated antenna position and the position of the star at peak received power, are normally distributed around their true value d. In other words the deviations v_i in (16) have a normal distribution with zero mean and variance $\sigma^2 = \sum_{i=1}^{n} v_i^2$. Making n measurements means selecting n samples of the ensemble d_i. For a sufficiently large number of samples (> 30) the mean of the samples is normally distributed with the same mean d but with a variance

$$s^2 = \frac{\sigma^2}{n-2} = \frac{\sum v_i^{\,2}}{n-2} \tag{24}$$

s is the error of the value d as determined by n measurements.

The error in d can be related to the errors of the elevation offset σ_E and azimuth offset σ_A by means of (16): this equation can be written as

$$v_i = -d_i + r_i \cos(\gamma i - \beta_i) = f\left[r_i(\Delta E, \Delta A),\ \gamma_i\ (\Delta E, \Delta A)\right]. \tag{25}$$

Linearization of (25) gives

$$v_i = -d_i + a_i\ \Delta E + b_i\ \Delta A \tag{26}$$

where

$$a_i = \frac{\partial f}{\partial \Delta E} = \cos\beta_i \tag{27}$$

$$b_i = \frac{\partial f}{\partial \Delta A} = \sin\beta_i\ \cos E_{1i}\ . \tag{28}$$

From the theory of regression analysis (Ref. 13) it is known that the variances of the best fitting values of the variables ΔE and ΔA are related to the variance of the observed variables d_i by

$$\sigma^2_{\Delta E} = s^2 \frac{\sum b_i^{\,2}}{\sum a_i^{\,2} \sum b_i^{\,2} - \sum a_i b_i} \tag{29}$$

$$\sigma^2_{\Delta A} = s^2 \frac{\sum a_i^{\,2}}{\sum a_i^{\,2} \sum b_i^{\,2} - \sum a_i b_i}$$

where s^2 is given by (24). These formulae are incorporated in the computer program at Appendix D; thus the program yields the best fitting values for the misalignments in elevation and azimuth and also their probable errors.

5. CALIBRATION OF SET-2

The experimental ground terminal SET-2 at the SHAPE Technical Centre provides tracking data of the NATO satellites for orbit determination. For this purpose the pointing accuracy of this terminal has been measured by the method described in this memorandum.

Between June and September 1972 a total of 36 recordings of Cassiopeia, Taurus, sun, and moon were taken. Figure 3 shows the positions of the radio sources at the times of the measurements. The result of the computer evaluation of the measurements is given as an example in Appendix C. The results are:

- Misalignment in elevation $\Delta E = 0.001^{\circ} \pm 0.010^{\circ}$
- Misalignment in azimuth $\Delta A = 0.009^{\circ} \pm 0.012^{\circ}$

from which it is concluded that within the accuracy of the measurement method there is no misalignment in elevation or azimuth. It should be kept in mind, that an antenna misalignment due to azimuth plane tilt was already subtracted. This was measured separately and is reported in Ref. 4. At azimuth and elevation angles associated with the NATO satellites this correction was 0.01° both in elevation and azimuth.

The error given above is derived from the scatter of the measurements. For comparison the error derived by considering the sources of uncertainties (Sections 4.1 and 4.2) is calculated below.

Pertinent equipment parameters of SET-2 are:

Antenna gain	: 51 dB
Antenna half-power beam width:	0.29° (elevation) 0.26° (azimuth)
System noise temperature	: 880°K
Radiometer time constant	: 4s

Therefore we have the following errors, due to the causes listed:

- Shift of the emission centre:

Cassiopeia	:	0°	(14 measurements)
Taurus	:	0.025°	(6 measurements)
Sun	:	0.025°	(13 measurements)
Moon	:	0.009°	(3 measurements)

Weighted mean : 0.014°

- Miscellaneous causes (Section 4.1) : 0.010°
- Correction for refraction : 0.015°
- Post-detection noise:

Sun and moon	0°	(16 measurements)
Cas. and Taurus	0.033°	(20 measurements)

Weighted mean : 0.018°

- Antenna asymmetry 0.030°
- Radiometer time constant correction:

Sun and moon	0°	(16 measurements)
Cas. and Taurus	0.004°	(20 measurements)

Weighted mean : 0.002°

- Time marks : 0.004°

---

rms sum : 0.042°

This is the probable error of one measurement. The error of the mean of 36 measurements is thus

$$\frac{0.042}{\sqrt{(36-2)}} = 0.007^\circ$$

This figure is in reasonable agreement with the errors derived from the scatter of the measurements, which were 0.010° and 0.012° respectively.

6. CONCLUSION

The error analysis shows that radio stars, the sun, and the moon are suitable for calibrating the read-out devices of a satellite ground terminal. A typical value for the accuracy is 0.006° for a 40-foot terminal. Calibration of terminals with a smaller antenna diameter would be less accurate because of the system noise. The calibration accuracy of terminals with a larger antenna diameter would not improve significantly because of the finite extension of the radio sources. The rms error can conveniently be derived from the scatter of the measurements.

The feasibility of the method has been demonstrated by applying it to the calibration of SET-2. With the accuracy limits of 0.01° no misalignments in elevation or azimuth have been found, apart from tilt of the azimuth plane, which was measured separately.

REFERENCES

1. A.D. Kuzmin and A.E. Salomonovich, "Radioastronomical methods of antenna measurements", Academic Press, New York and London, 1966.

2. "The astronomical ephemeris", Her Majesty's Stationary Office, London, published yearly.

3. C.E. Ewing and M.M. Mitchell, "Introduction to geodesy", American Elsevier Publ. Comp. Inc., New York, 1970.

4. A. Wallrabe, "Acquisition and recording of tracking data at the satellite ground terminal SET-2", STC Technical Memorandum TM-375, May 1973 (NATO Unclassified).

5. "Explanatory supplement to the astronomical ephemeris", London, Her Majesty's Stationary Office, 1961.

6. C.C.I.R. Documents of the XIIth Plenary Assembly, New Delhi, 1970, Vol. II, Part I, Recommendation 453 and Report 232-2.

7. K. Takahashi, "Atmospheric error in range and range-rate measurements between a ground station and an artificial satellite", IEEE Trans. AES-6 (1970), No. 6, p. 770 - 779.

8. D.A. Guidice and J.P. Castelli, "The use of extra-terrestrial radio sources in the measurement of antenna parameters", IEEE Trans. AES-7 (1971), No. 2, p. 226 - 234.

9. P. Mezger, "Verzerrung radioastronomischer Beobachtungen durch Antennencharakteristik und Tiefpass", Zeitschrift fur Instrumentenkunde 66 (1958), No. 11, p. 219 - 227.

10. W. Graf, R.N. Bracewell, J.H. Denter, and J.S. Rutherford, "The sun as a test source for boresight calibration of microwave antennas", IEEE Trans. AP-19 (1971), No. 5, p. 606 - 613.

11. R.W. Roig and G.E. Roberts, "Radar measurement errors attributable to atmospheric refractivity", Advanced Research Projects Agency, The Pentagon, Washington D.C., USA, April 1969, Contract No. DAH01-67-C-1334.

12. A. Heilmann, "Antennen", Vol. 2, BI Hochschultaschenbücher Bd. 534/534a, Mannheim, 1970.

13. E. Hardtwig, "Fehler - und Ausgleichsrechnung", BI Hochschultaschenbücher Bd. 262/262a, Mannheim, 1968.

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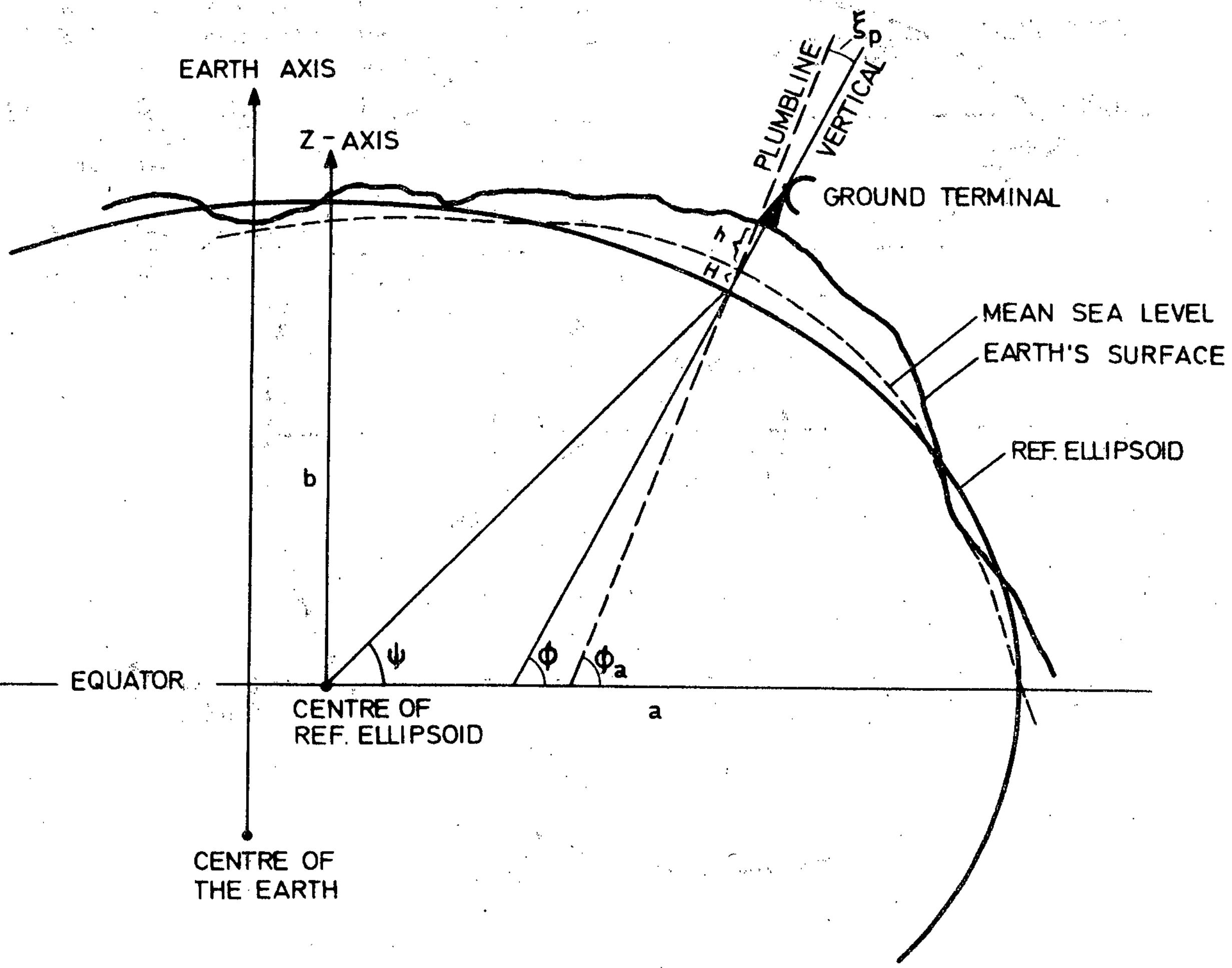

Fig. 1 Location of the ground terminal

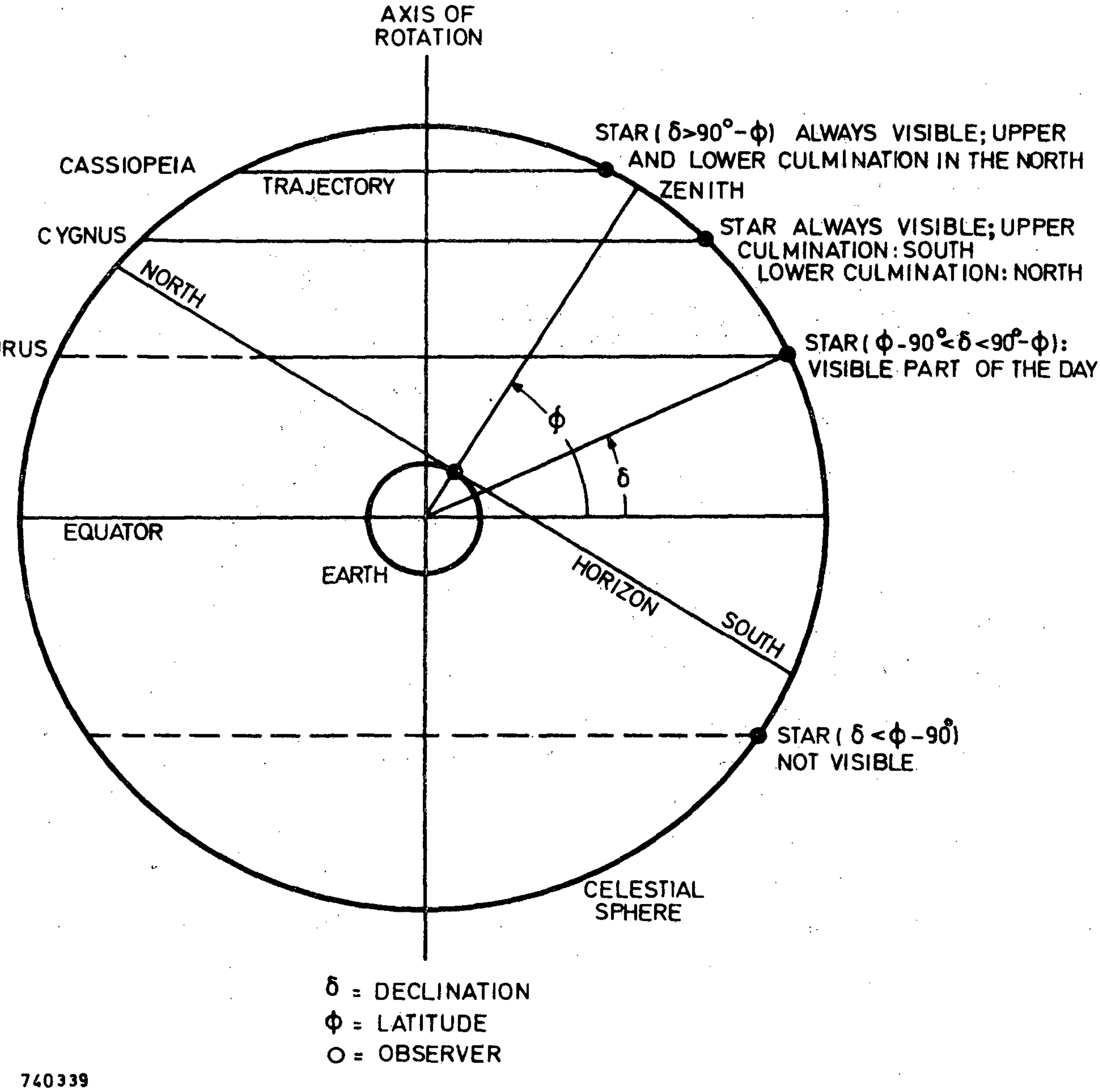

Fig. 2 Visibility of stars of various declinations

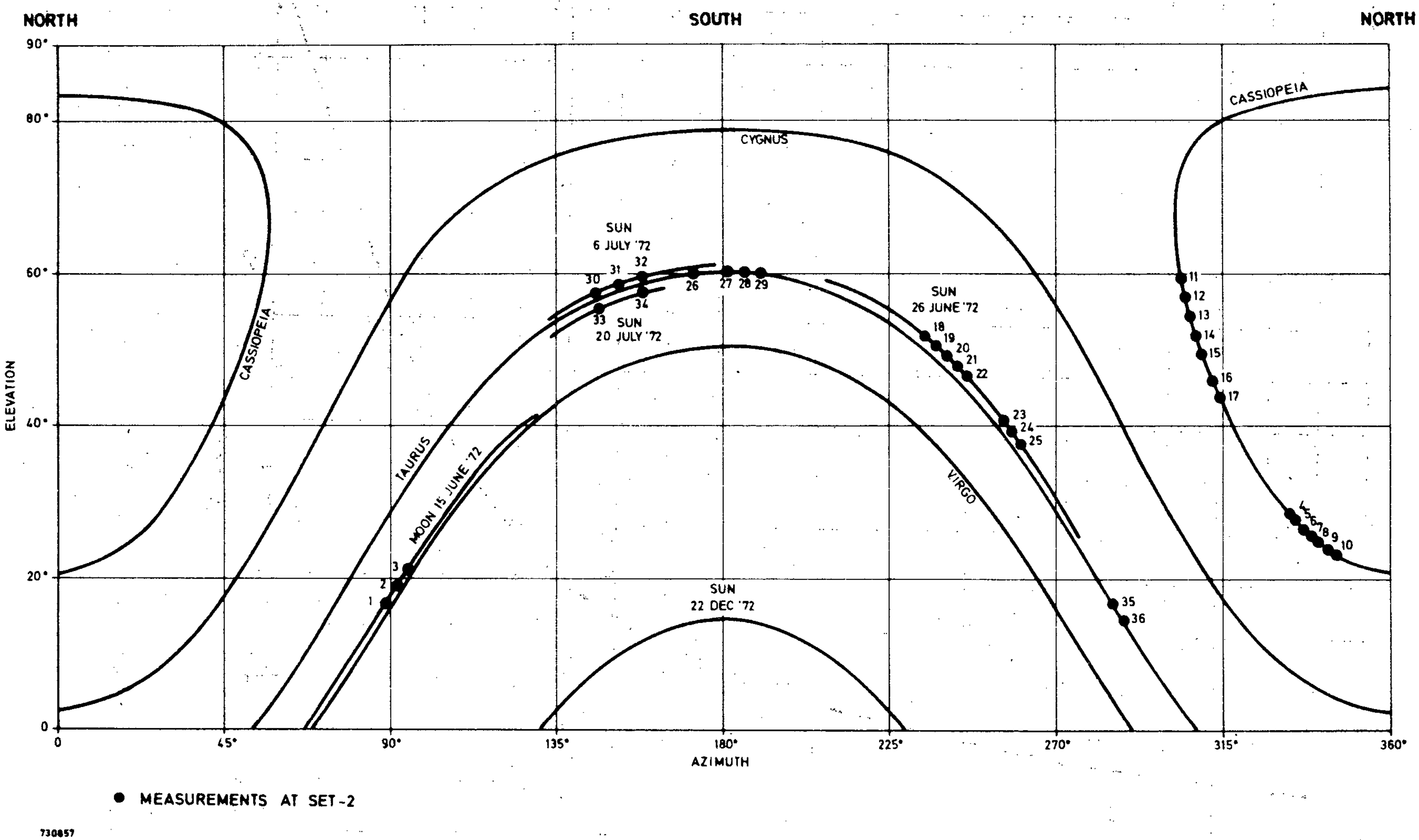

Fig. 3 Positions of radio sources in the sky for SET-2

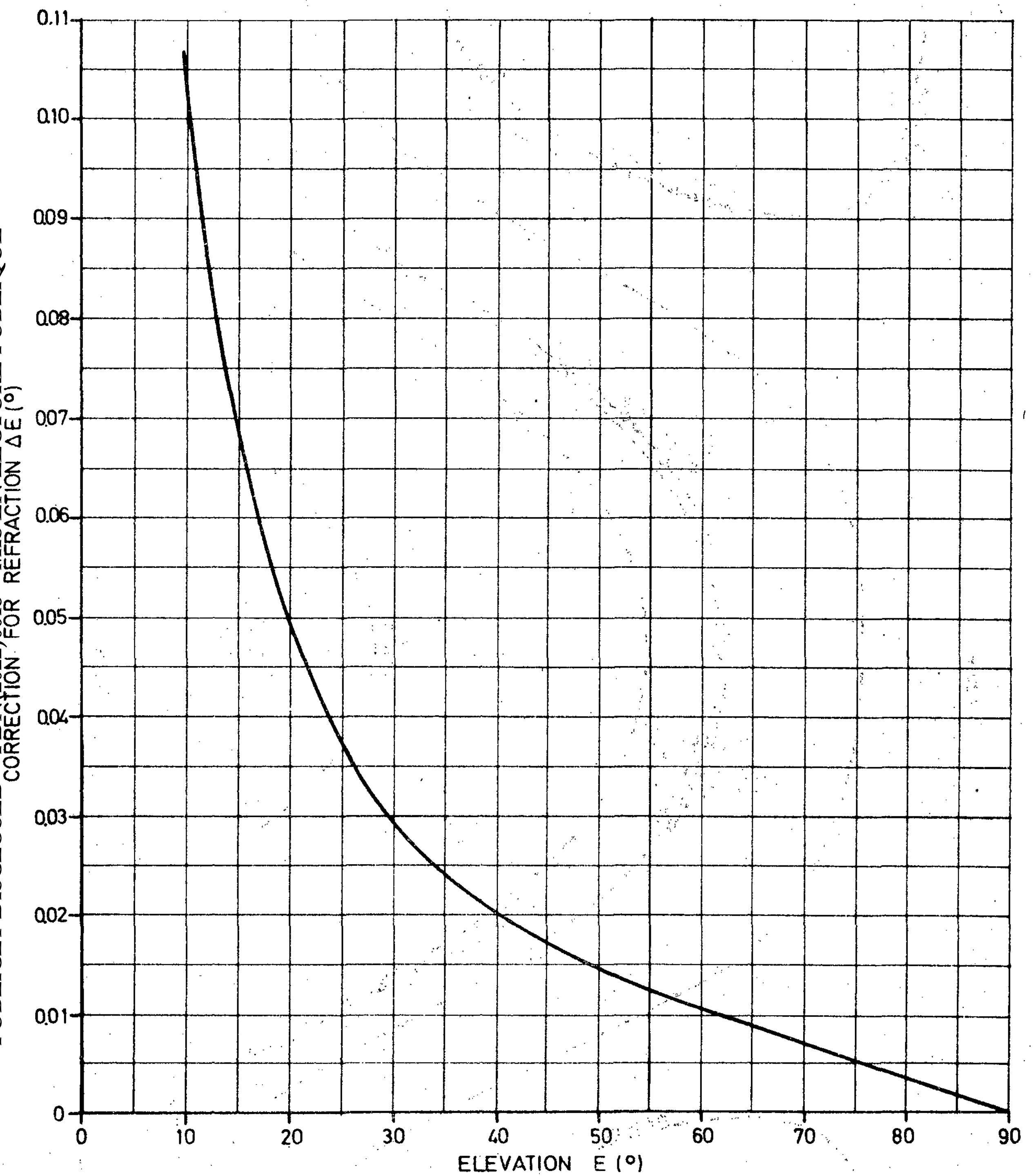

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Fig. 4 Correction of the elevation angle for refraction

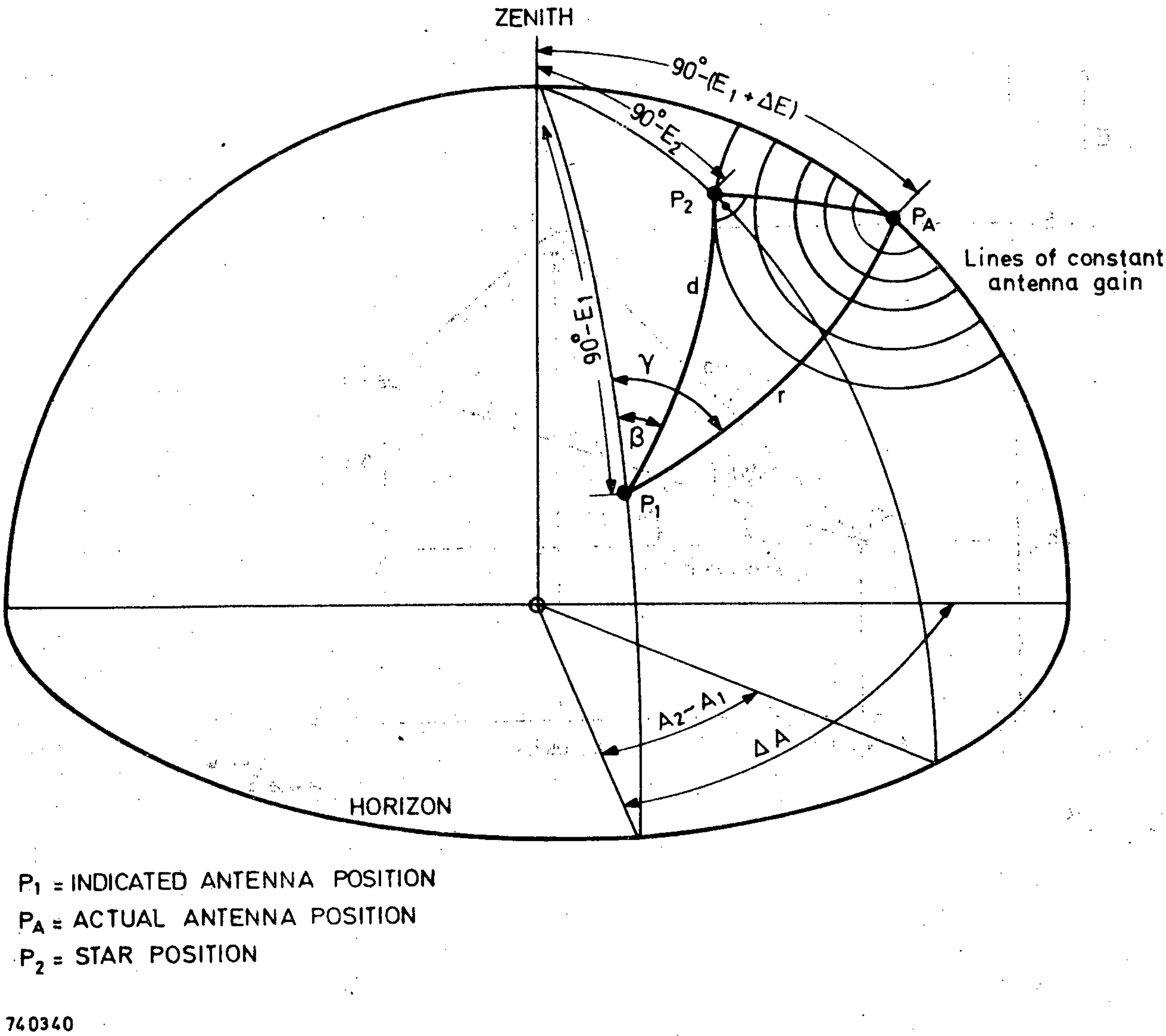

Fig. 5 Relations between antenna and star positions

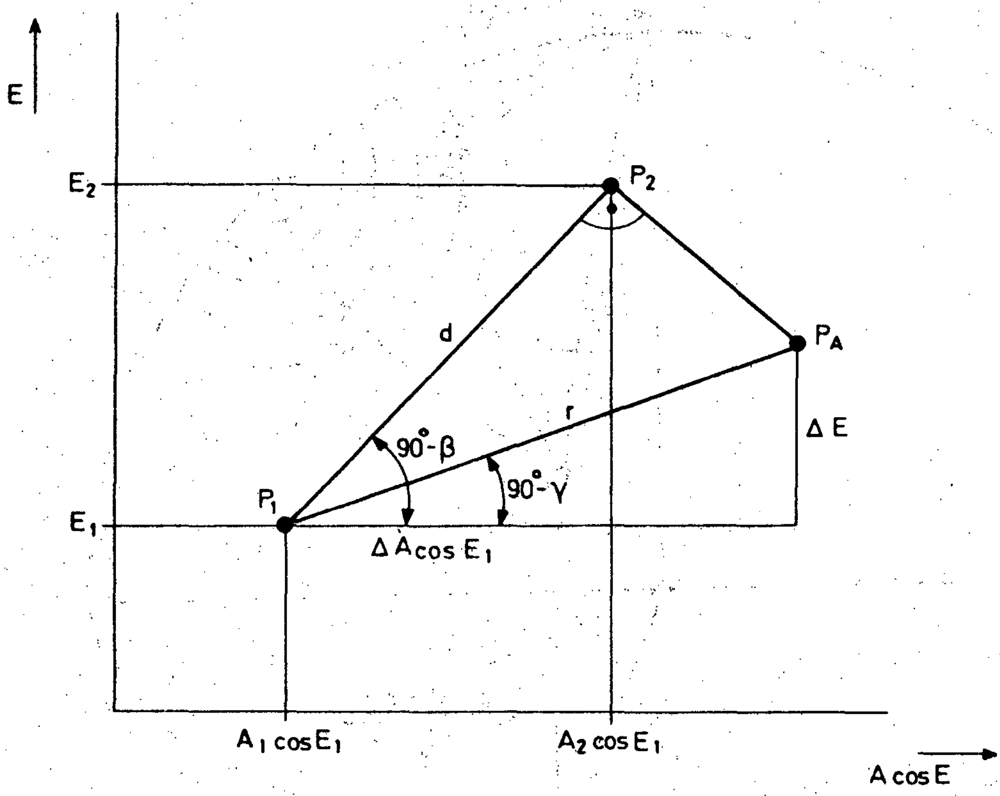

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Fig. 6 Approximation of the spherical triangle (Fig. 5) by a plane triangle

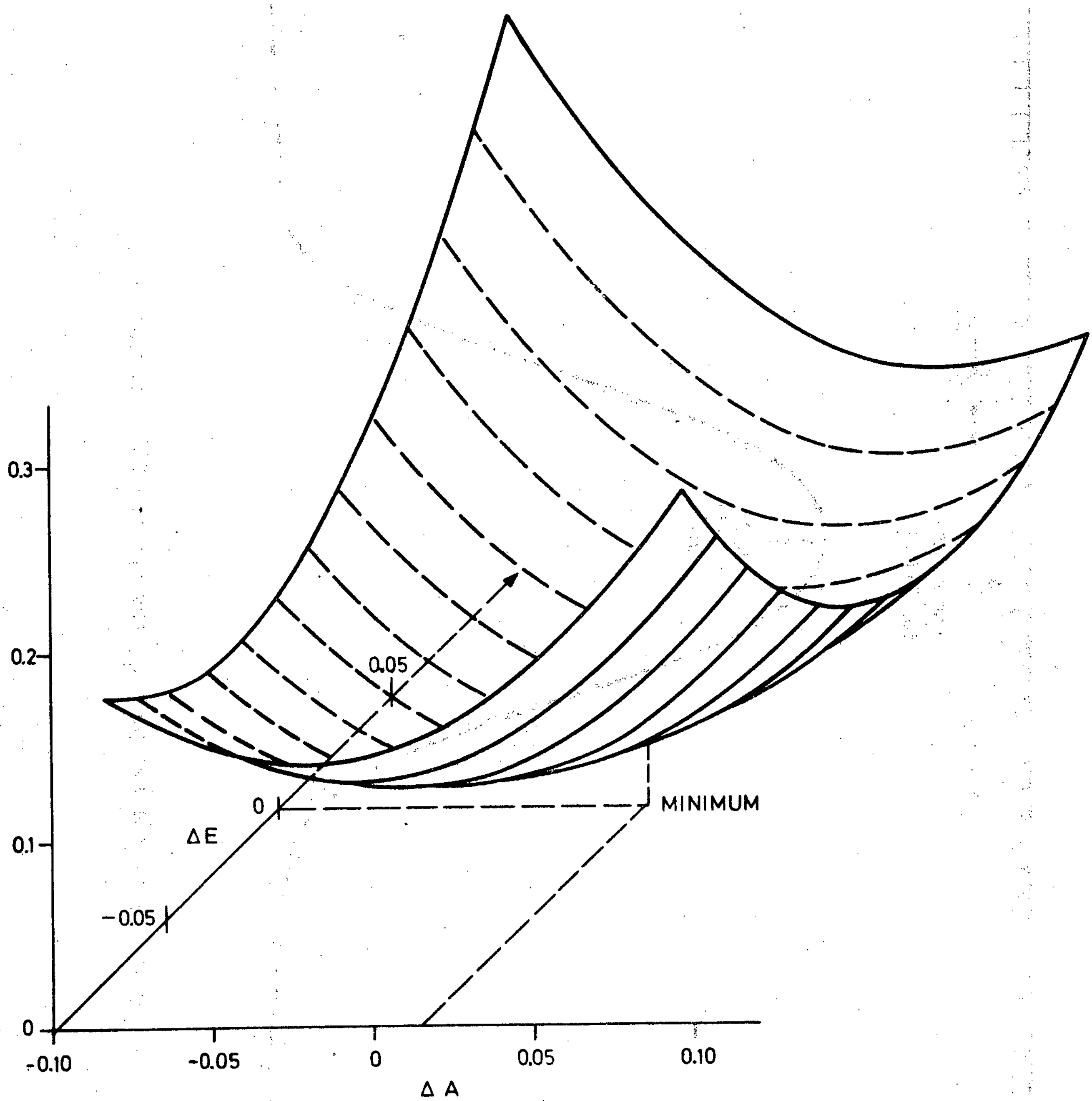

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Fig. 7 F(ΔE, ΔA) as function of the elevation and azimuth misalignments ΔE and ΔA

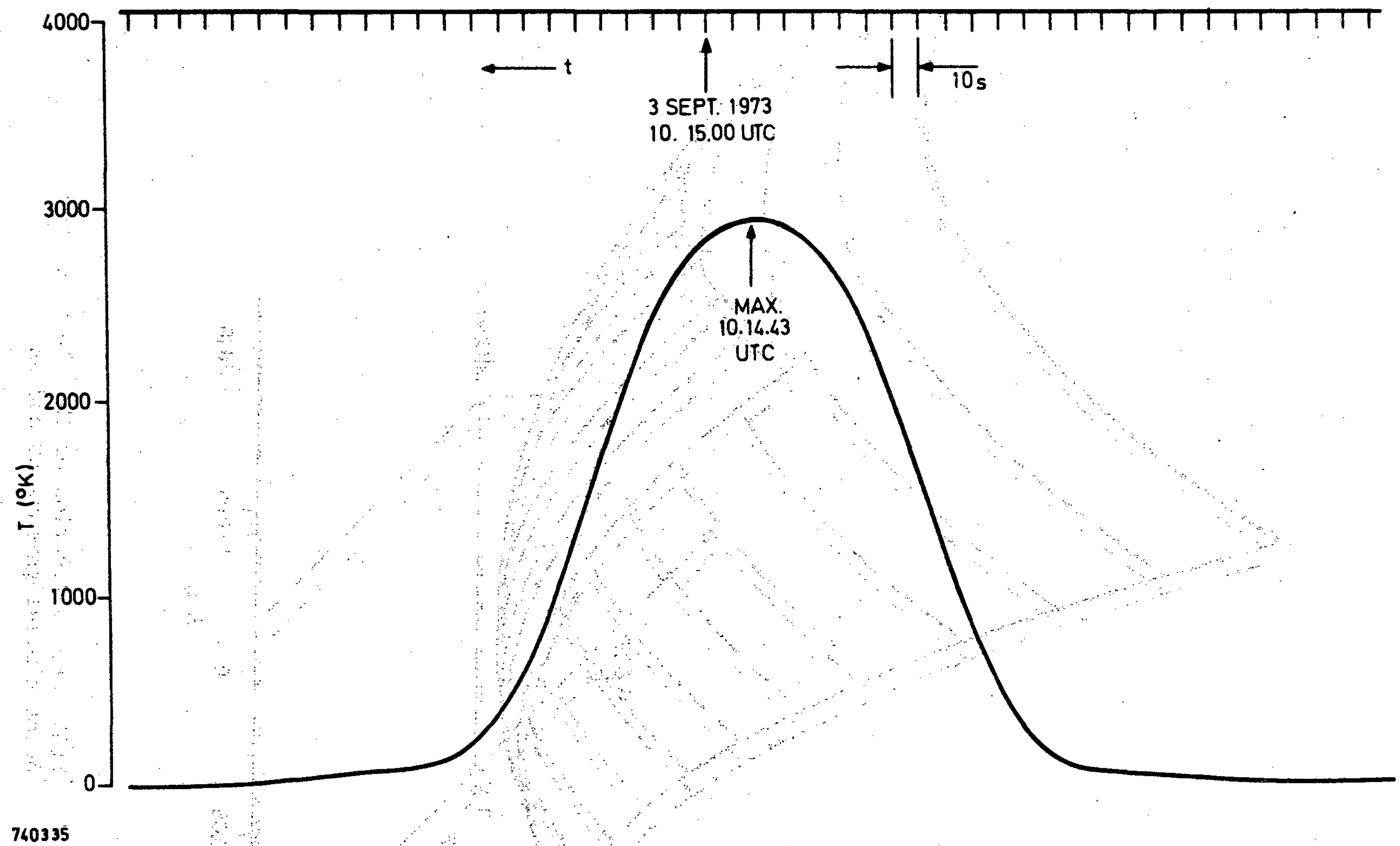

Fig. 8 Recording of the sun passing through the SET-2 antenna

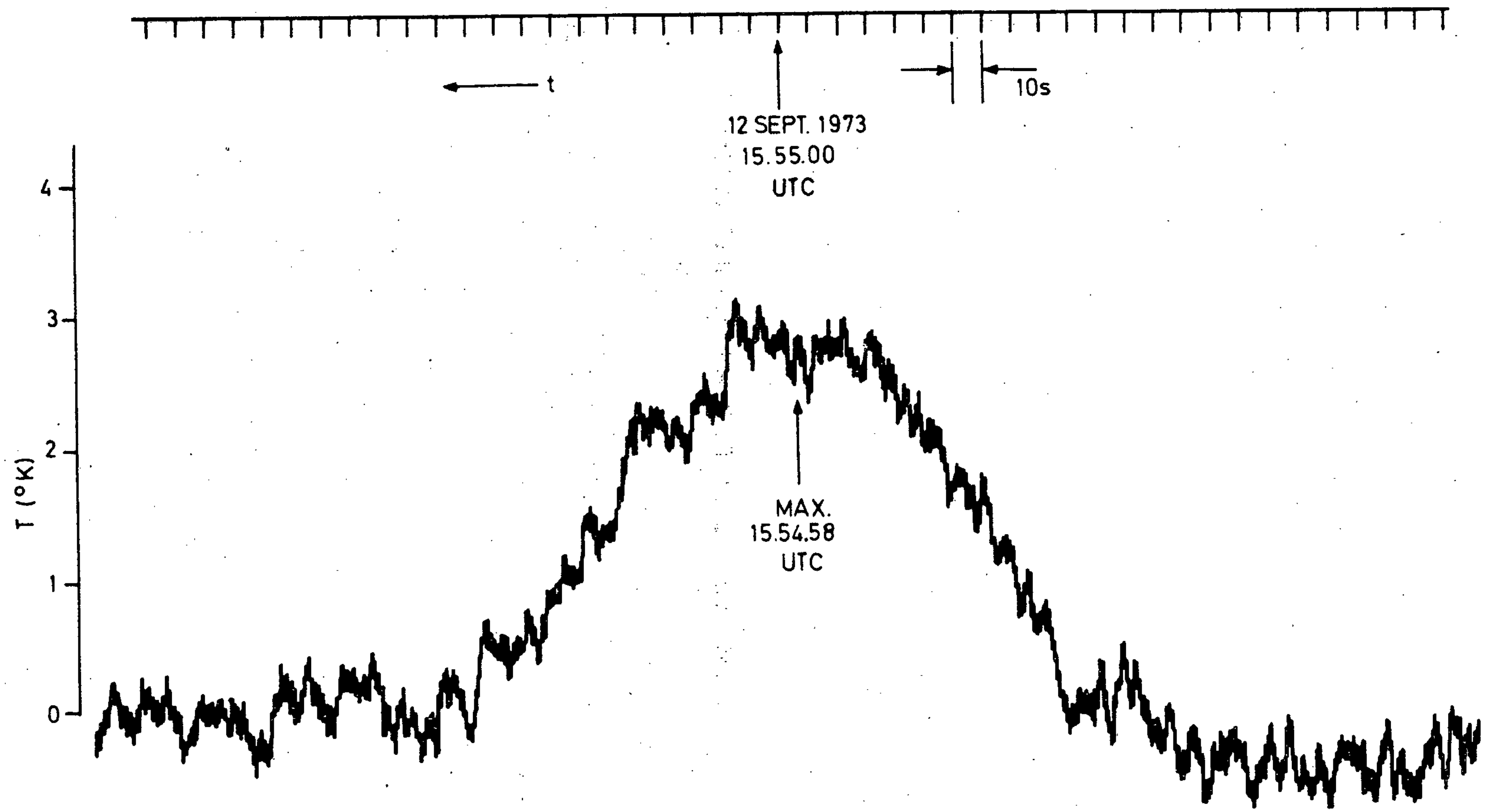

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Fig. 9 Radiometer recording of Cassiopeia A at SET-2

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APPENDIX A

CALCULATION OF ELEVATION AND AZIMUTH OF FIXED STARS

The formulae used are

$$H = H_\gamma + \lambda - \alpha + UT\ 0.997\ 269\ 5664 \tag{A1}$$

where

H = hour angle of star

H_γ = hour angle of First Point of Aries at 0 UT

λ = longitude of observer (positive to the east)

α = right ascension of star

UT = Universal Time

and $$\sin E = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H \tag{A2}$$

where A = azimuth

E = elevation

ϕ = latitude of observer

δ = declination of star

and $$\tan A' = \frac{-\cos\delta \sin H}{\sin\delta \cos\phi - \cos\delta \sin\phi \cos H} \tag{A3}$$

$$A = 360^\circ + A' \quad \text{for } A' < 0$$

$$A = A' \quad \text{for } A' \geq 0$$

If α_o and δ_o are given at Epoch 1950.0 then α' and δ' for the beginning of the current year are calculated by

$$q = \sin\theta \left[\tan\delta_o + \cos(\alpha_o + \xi_o) \tan \frac{1}{2}\theta\right] \tag{A4}$$

$$\alpha' = \alpha_o + \xi_o + z + \arctan \frac{q \sin(\alpha_o + \xi_o)}{1 - q \cos(\alpha_o + \xi_o)} \tag{A5}$$

$$\delta' = \delta_o + 2 \arctan \left\{ \tan \frac{1}{2}\theta \frac{\cos \frac{1}{2}\left[\alpha' + (\alpha_o + \xi_o - z)\right]}{\cos \frac{1}{2}\left[\alpha' - (\alpha_o + \xi_o + z)\right]} \right\} \tag{A6}$$

The transfer from the beginning of the year to the current date is calculated by

$$\alpha = \alpha' + m + n \sin\alpha' \tan\delta' \tag{A7}$$

$$\delta = \delta' + n \cos\alpha' \tag{A8}$$

The constants ξ_o, z, θ, m, and n in (A4) to (A8) may be found in the AE. For the years 1972 to 1975 they are as follows:

	ξ_o	z	θ	m	n
1972	8'27".10	8'27".14	7'20".92	$3^s.07368$	$1^s.33605$
1973	8'50".20	8'50".15	7'40".96	$3^s.07370$	$1^s.33604$
1974	9'13".25	9'13".21	8'01".00	$3^s.07372$	$1^s.33603$
1975	9'36".31	9'36".26	8'21".04	$3^s.07374$	$1^s.33603$

Right ascension and declination for some radio stars (Epoch 1950.0)

Star	α_o	δ_o
Taurus A	82.87917	21.98333
Cygnus A	299.43333	40.62333
Cassiopeia A	350.29583	58.54667

HP 9100 B desk calculator program

User instructions

Set	:	DEGREES
Press	:	END
Enter program	:	Magnetic card side A followed by side B
Press	:	END
Press	:	CONTINUE
Display	:	$\boxed{1 \quad 0 \quad 0}$
Enter	:	φ (degs.) → y λ (degs.) → x

Press : CONTINUE
Display : $\boxed{2 \quad 0 \quad 0}$
Enter : α_o (degs.) → y
δ_o (degs.) → x

Press : CONTINUE
Display : $\boxed{3 \quad 0 \quad 0}$
Enter : ξ'_o → y
ξ''_o → x

Press : CONTINUE
Display : $\boxed{4 \quad 0 \quad 0}$
Enter : z' → y
z" → x

Press : CONTINUE
Display : $\boxed{5 \quad 0 \quad 0}$
Enter θ' → y
θ'' → x

Press : CONTINUE
Display : $\boxed{6 \quad 0 \quad 0}$
Enter m(s) → y
n(s) → x

Press : CONTINUE
Display : $\boxed{7 \quad 0 \quad 0}$
Enter : Day (1 Jan. = 1) → x
Press : CONTINUE
Display : $\boxed{8 \quad 0 \quad 0}$
Enter : H_γ (hours) → z
H_γ (min) → y
H_γ (s) → x

Display : $\boxed{9 \quad 0 \quad 0}$
Enter : UT (hours) → z
UT (min) → y
UT (s) → x

Press : CONTINUE
Display : $\boxed{0 \quad A \quad E}$

New time?
Yes; Press: CONTINUE

Title: Elevation and azimuth of fixed stars

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
01	1	01		display	
02	STOP	41	1	0	0
03	x→	23		enter	
04	d	17	λ	φ	
05	y→	40			
06	c	16			
07	CLEAR	20			
08	2	02		display	
09	STOP	41	2	0	0
0a	y→	40		enter	
0b	b	14	δ_0	α_0	
0c	x→	23			
0d	a	13			
10	CLEAR	20			
11	3	03		display	
12	STOP	41	3	0	0
13	↑	27		enter	
14	6	06	ε_0''	ε_0'	
15	0	00			
16	÷	35			
17	↓	25			
18	+	33			
19	6	06			
1a	0	00			
1b	÷	35			
1c	b	14			
1d	+	33			
20	y→	40			
21	−	34			
22	f	15			
23	CLEAR	20			
24	4	04			
25	STOP	41		display	
26	↑	27	4	0	0
27	6	06		enter	
28	0	00	z''	z'	
29	÷	35			
2a	↓	25			
2b	+	33			
2c	6	06			
2d	0	00			

Step	Key	Code	Display x	Display y	Display z
30	÷	35			
31	x←	67			
32	−	34			
33	f	15			
34	+	33			
35	y→	40			
36	−	34			
37	e	12			
38	CLEAR	20		display	
39	5	05	5	0	0
3a	STOP	41		enter	
3b	↑	27	θ''	θ'	
3c	6	06			
3d	0	00			
40	÷	35			
41	↓	25			
42	+	33			
43	6	06			
44	0	00			
45	÷	35			
46	↓	25			
47	↑	27			
48	sin	70			
49	ACC+	60			
4a	2	02			
4b	÷	35			
4c	↓	25			
4d	tan	71			
50	x→	23			
51	e	12			
52	↑	27			
53	x←	67			
54	−	34			
55	f	15			
56	cos	73			
57	×	36			
58	a	13			
59	tan	71			
5a	+	33			
5b	f	15			
5c	×	36			
5d	y→	40			

Step	Key	Code	Display x	Display y	Display z
60	f	15			
61	x←	67			
62	−	34			
63	f	15			
64	↑	27			
65	Roll↓	31			
66	sin	70			
67	×	36			
68	Roll↑	22			
69	cos	73			
6a	x⇄y	30			
6b	f	15			
6c	×	36			
6d	1	01			
70	x⇄y	30			
71	−	34			
72	↓	25			
73	÷	35			
74	↓	25			
75	arc	72			
76	tan	71			
77	↑	27			
78	x←	67			
79	−	34			
7a	e	12			
7b	+	33			
7c	y→	40			
7d	b	14			

Storage	
f	
e	
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b	
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5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

Title: Elevation and azimuth of fixed stars

Step	Key	Code	Display x	Display y	Display z
80	−	34			
1	2	02			
2	÷	35			
3	↑	27			
4	↓	25			
5	x←	67			
6	−	34			
7	f	15			
8	+	33			
9	↓	25			
a	cos	73			
b	x⇄y	30			
c	cos	73			
d	÷	35			
90	e	12			
1	×	36			
2	↓	25			
3	arc	72			
4	tan	71			
5	↑	27			
6	2	02			
7	×	36			
8	a	13			
9	+	33			
a	GOTO	44			
b	−	34			
c	0	00			
d	0	[illegible]			
a0	y→	40			
1	a	13			
2	CLEAR	20			
3	6	06			
4	STOP	41		display	
5	ACC+	60	6	0	0
6	CLEAR X	37		enter	
7	↑	27	m^1	m^3	
8	↑	27			
9	7	07			
a	STOP	41		display	
b	Roll ↓	31	7	0	0
c	RCL	61		enter	
d	x⇄y	30	DAY		
10	Roll ↑	22			
1	×	36			
2	x⇄y	30			
3	Roll ↓	31			
4	×	36			
5	8	10			
6	7	07			
7	6	06			
8	ENTER EXP	26			
9	4	04			
a	÷	35			
b	x⇄y	30			
c	Roll ↓	31			
d	÷	35			
20	↓	25			
1	y→	40			
2	e	12			
3	x⇄y	30			
4	↑	27			
5	b	14			
6	sin	70			
7	×	36			
8	a	13			
9	tan	71			
a	×	36			
b	↓	25			
c	+	33			
d	b	14			
30	+	33			
1	y→	40			
2	b	14			
3	cos	73			
4	y⇄	24			
5	e	12			
6	×	36			
7	a	13			
8	+	33			
9	y→	40			
a	a	13			
b	CLEAR	20			
c	8	10			
d	STOP	41			
40	GOTO	44		display	
1	SUB	77	8	0	0
2	−	34		enter	
3	c	16	H_g^{sec}	H_g^{m}	H_g^{h}
4	c	16			
5	y→	40			
6	−	34			
7	f	15			
8	CLEAR	20			
9	9	11		display	
a	STOP	41	9	0	0
b	GOTO	44		enter	
c	SUB	77	T^{sec}	T^{m}	T^{h}
d	−	34			
50	c	16			
1	c	16			
2	°	21			
3	9	11			
4	9	11			
5	7	07			
6	2	02			
7	6	06			
8	9	11			
9	5	05			
a	6	06			
b	6	06			
c	4	04			
d	÷	35			

Storage	
f	
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c	
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a	
9	
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7	
6	
5	
4	
3	
2	
1	
0	

Title: Elevation and azimuth of fixed stars

Step	Key	Code	Display x	Display y	Display z
6 0	b	14			
1	−	34			
2	d	17			
3	+	33			
4	x ←	67			
5	−	34			
6	f	15			
7	+	33			
8	↓	25			
9	x →	23			
a	e	12			
b	cos	73			
c	↑	27			
d	a	13			
7 0	cos	73			
1	×	36			
2	c	16			
3	cos	73			
4	×	36			
5	c	16			
6	sin	70			
7	↑	27			
8	a	13			
9	sin	70			
a	×	36			
b	↓	25			
c	+	33			
d	↓	25			
8 0	arc	72			
1	sin	70			
2	x →	23			
3	f	15			
4	e	12			
5	sin	70			
6	↑	27			
7	a	13			
8	cos	73			
9	CHG SIGN	32			
a	×	36			
b	y →	40			
c	−	34			
d	e	12			

Step	Key	Code	Display x	Display y	Display z
9 0	a	13			
1	sin	70			
2	↑	27			
3	c	16			
4	cos	73			
5	×	36			
6	a	13			
7	cos	73			
8	↑	27			
9	c	16			
a	sin	70			
b	×	36			
c	e	12			
d	cos	73			
a 0	×	36			
1	↓	25			
2	−	34			
3	↓	25			
4	y ⇄	24			
5	−	34			
6	e	12			
7	TO POLAR	62			
8	CLEAR X	37			
9	IF x<y	52			
a	b	14			
b	2	02			
c	3	03			
d	6	06			
b 0	0	00			
1	x ⇄ y	30			
2	+	33			
3	f	15			
4	x ⇄ y	30			
5	↑	27			
6	CLEAR X	37			
7	STOP	41		display	
8	GO TO	44	0	A	E
9	4	04			
a	8	10			
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
c 0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b			Subroutine		
c	ACC +	60	sec	min	h
d	6	06			
d 0	0	00			
1	÷	35			
2	ROLL ↓	31			
3	+	33			
4	f	15			
5	ROLL ↑	22			
6	÷	35			
7	÷	35			
8	↓	25			
9	+	33			
a	1	01			
b	5	05			
c	×	36		degs	
d	SUB/R	77			

Storage

APPENDIX B

CALCULATION OF ELEVATION AND AZIMUTH OF THE SUN

The formulae used are

$$\alpha = \alpha_o + f\ \Delta\alpha_o \qquad \text{(B1)}$$

$$\delta = \delta_o + f\ \Delta\delta_o \qquad \text{(B2)}$$

where

$$\left.\begin{array}{lcl} \alpha_o & = & \text{right ascension of sun} \\ \delta_o & = & \text{declination of sun} \end{array}\right\} \text{ at } 0^h \text{ Ephemeris Time (ET)}$$

ET = $UT + \Delta T$, see table below

$\Delta\alpha_o$ = daily rate of right ascension

$\Delta\delta_o$ = daily rate of declination

f = fraction of day

$$H = H_\gamma + \lambda - \alpha + UT/0.997\ 269\ 5664 \qquad \text{(B3)}$$

where

H = hour angle

H_γ = hour angle of First Point of Aries at 0 UT

λ = longitude of observer

ϕ = latitude of observer

Correction for parallax is applied as follows:

$$\alpha_{corr} = \alpha + 8''.8 \cos\phi \sin H \sec\delta \qquad \text{(B4)}$$

$$H_{corr} = H - 8''.8 (\sin\phi \cos\delta - \cos\phi \cosh \sin\delta). \qquad \text{(B5)}$$

Then elevation E and azimuth A are calculated in the same way as in Appendix A.

The correction ΔT may be found in the AE. For the years 1972 to 1975 it is as follows:

	Jan 72	July 72	Jan 73	July 73	Jan 74	July 74	Jan 75
ΔT(s)	42.23	42.8	43.4	44.0	44.6	45.2	45.8

HP 9100 B desk calculator program

User instructions

Set:	DEGREES
Press:	END
Enter program:	Side A followed by side B
Press:	END
→ Press:	CONTINUE
Display :	$\boxed{1 \quad 0 \quad 0}$
Enter:	ϕ (latitude in degs.) → y
	λ (longitude in degs.) → x (positive to the east)
Press:	CONTINUE
Display:	$\boxed{2 \quad 0 \quad 0}$
Enter:	H_γ (hours) → z
	H_γ (mins) → y
	H_γ (seconds and fractional seconds) → x
Press:	CONTINUE
Display:	$\boxed{3 \quad 0 \quad 0}$
	α_o (hours) → z
	α_o (mins) → y
	α_o (seconds and fractional seconds) → x
Press:	CONTINUE
Display	$\boxed{4 \quad 0 \quad 0}$
Enter:	$\Delta\alpha_o$ (seconds and fract. seconds) → x
Press:	CONTINUE
Display:	$\boxed{5 \quad 0 \quad 0}$
	δ_o (degrees) → z
	δ_o (min. of arc) → y
a	δ_o (seconds of arc and fract.) → x

a

Press: CONTINUE

Display: | 6 0 0 |

Enter: $\Delta\delta_o$ (seconds or arc and fract.) → x

Press: CONTINUE

Display: | 7 0 0 |

Enter: UT (hours) → z

UT (minutes) → y

UT (seconds and fract.) → x

Press: CONTINUE

Display:

Elevation	→	z
Azimuth	→	y
0	→	x

New time instant
Same day ?

yes

no

Press: SET FLAG

Title: Azimuth and elevation of the sun

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20		display	
01	1	01	1	0	0
02	STOP	41		enter	
03	x→	23	λ	φ	
04	d	17			
05	y→	40		store	
06	c	16			
07	CLEAR	20			
08	2	02		display	
09	STOP	41	2	0	0
0a	GOTO	44		enter	
0b	SUB	77	H_γ^s	H_γ^m	H_γ^h
0c	−	34			
0d	9	11	Convert	to	degs
10	c	16	and	store	
11	y→	40			
12	b	14			
13	CLEAR	20			
14	3	03		display	
15	STOP	41	3	0	0
16	GOTO	44		enter	
17	SUB	77	a_0^s	a_0^m	a_0^h
18	−	34			
19	9	11			
1a	c	16	Convert	to	degs
1b	y→	40	and	store	
1c	−	34			
1d	f	15			
20	CLEAR	20			
21	4	04		display	
22	STOP	41	4	0	0
23	GOTO	44		enter	
24	SUB	77	Δa^s		
25	−	34			
26	9	11			
27	c	16	convert	to	degs
28	y→	40	and	store	
29	−	34			
2a	e	12			
2b	CLEAR	20			
2c	5	05		display	
2d	STOP	41	5	0	0

Step	Key	Code	Display x	Display y	Display z
30	ACC+	60		enter	
31	6	06	δ''	δ'	δ°
32	0	00			
33	÷	35			
34	Roll↓	31			
35	+	33			
36	f	15	Convert	to	degs
37	Roll↑	22	and	store	
38	÷	35			
39	÷	35			
3a	↓	25			
3b	+	33			
3c	y→	40			
3d	−	34			
40	d	17			
41	CLEAR	20			
42	6	06		display	
43	STOP	41	6	0	0
44	↑	27		enter	
45	6	06	Δδ''		
46	0	00			
47	÷	35			
48	÷	35			
49	y→	40	Convert	to	degs.
4a	−	34	and	store	
4b	c	16			
4c	CLEAR	20			
4d	7	07		display	
50	STOP	41	7	0	0
51	GOTO	44		enter	
52	SUB	77	UT^s	UT^m	UT^h
53	−	34			
54	9	11	Convert	to	degs
55	c	16	and	store	
56	y→	40			
57	−	34			
58	b	14			
59	4	04	ET − UT		
5a	3	03			
5b	↑	27			
5c	2	02			
5d	4	04			

Step	Key	Code	Display x	Display y	Display z
60	0	00			
61	÷	35	Calculate		
62	↓	25	Ephem.	time	
63	+	33	and	convert	
64	3	03	to	fraction	
65	6	06	of	day	
66	0	00			
67	÷	35			
68	↑	27			
69	↓	25			
6a	x←	67			
6b	−	34			
6c	e	12	Calculate	α	
6d	×	36	at ET	and	store
70	x←	67			
71	−	34			
72	f	15			
73	+	33			
74	y→	40			
75	f	15			
76	↓	25			
77	x←	67			
78	−	34	Calculate	δ	
79	c	16	at ET	and	store
7a	×	36			
7b	x←	67			
7c	−	34			
7d	d	17			

Storage

	+	−
f	δ'', α, x	a_0
e	δ', δ	Δα
d	λ	δ_0
c	φ	Δδ
b	H_γ	UT, E
a	H	
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

HEWLETT · PACKARD

HEWLETT-PACKARD

Title Elevation and azimuth of the sun

Step	Key	Code	Display x	Display y	Display z
80	+	33			
81	y→	40			
82	e	12			
83	y↻	24			
84	−	34			
85	b	14			
86	·	21	calculate		
87	9	11	time in sideral		
88	9	11	units and H.		
89	7	07	store		
8a	2	02			
8b	6	06			
8c	9	11			
8d	5	05			
90	6	06			
91	6	06			
92	4	04			
93	÷	35			
94	f	15			
95	−	34			
96	d	17			
97	+	33			
98	b	14			
99	+	33			
9a	GOTO	44			
9b	−	34			
9c	0	00			
9d	0	00			
00	y→	40			
01	a	13			
02	↓	25			
03	sin	70			
04	↑	27			
05	c	16			
06	cos	73	Calculate		
07	×	36	correction Δα		
08	e	12	due to parallax		
09	cos	73	and subtract		
0a	÷	35	from h		
0b	8	10			
0c	·	21			
0d	8	10			
10	×	36			
11	6	06			
12	0	00			
13	÷	35			
14	÷	35			
15	↓	25			
16	y↻	24			
17	a	13			
18	−	34			
19	y→	40			
1a	a	13			
1b	c	16			
1c	sin	70			
1d	↑	27			
20	e	12			
21	cos	73	calculate		
22	×	36	correction Δδ		
23	e	12	due to parallax		
24	sin	70	and add to δ		
25	↑	27			
26	a	13			
27	cos	73			
28	×	36			
29	c	16			
2a	cos	73			
2b	×	36			
2c	↓	25			
2d	−	34			
30	8	10			
31	·	21			
32	8	10			
33	×	36			
34	6	06			
35	0	00			
36	÷	35			
37	÷	35			
38	ACC+	60			
39	a	13			
3a	cos	73			
3b	↑	27			
3c	e	12			
3d	cos	73			
40	×	36			
41	c	16			
42	cos	73			
43	×	36	calculate F		
44	e	12	and store		
45	sin	70			
46	↑	27			
47	c	16			
48	sin	70			
49	×	36			
4a	↓	25			
4b	+	33			
4c	↓	25			
4d	arc	72			
50	sin	70			
51	x→	23			
52	−	34			
53	b	14			
54	c	16			
55	cos	73	calculate A		
56	↑	27	and arrange		
57	e	12	0 ≤ A < 360°		
58	sin	70			
59	×	36			
5a	e	12			
5b	cos	73			
5c	↑	27			
5d	c	16			

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Title Elevation and azimuth of the sun

Step	Key	Code	Display x	Display y	Display z
6 0	sin	70			
1	x	36			
2	a	13			
3	cos	73			
4	x	36			
5	↓	25			
6	−	34			
7	y→	40			
8	f	15			
9	a	13			
a	sin	70			
b	↑	27			
c	e	12			
d	cos	73			
7 0	CHG SIGN	32			
1	x	36			
2	f	15			
3	TO POLAR	62			
4	CLEAR X	37			
5	IF x≤y	52			
6	7	07			
7	c	16			
8	3	03			
9	6	06			
a	0	00			
b	+	33			
c	↓	25			
d	y⇄	24			
8 0	−	34			
1	b	14			
2	↑	27			
3	CLEAR X	37		display	
4	STOP	41	0	A	E
5	IF FLAG	43			
6	GOTO	44			
7	+	33			
8	0	00			
9	7	07			
a	GOTO	44			
b	+	33			
c	4	04			
d	c	16			

Step	Key	Code	Display x	Display y	Display z
9 0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b			Subroutine		
c	ACC+	60	sec	min	h
d	6	06			
a 0	0	00			
1	÷	35			
2	Roll ↓	31			
3	+	33			
4	f	15			
5	Roll ↑	22			
6	÷	35			
7	÷	35			
8	↓	25			
9	+	33			
a	1	01			
b	5	05			
c	x	36		degr.	
d	SUB/RET	77			

Storage

APPENDIX C

CALCULATION OF ELEVATION AND AZIMUTH OF THE MOON

The formulae used are

$$\alpha = \alpha_o + g\ \Delta\alpha_o \tag{C1}$$

$$\delta = \delta_o + g\ \Delta\delta_o \tag{C2}$$

where

$\left.\begin{array}{l}\alpha_o = \text{right ascension of moon} \\ \delta_o = \text{declination of moon}\end{array}\right\}$ at the preceding full hour

$\Delta\alpha_o$ = hourly rate of right ascension

$\Delta\delta_o$ = hourly rate of declination

g = fraction of the hour in Ephemeris Time (ET = UT + ΔT, see Table in Appendix B)

$$H = H_\gamma + \lambda - \alpha + UT/0.997\ 269\ 566\ 4 \tag{C3}$$

where

H = hour angle

H_γ = hour angle of First Point of Aries at 0 UT

λ = longitude of observer (positive to the east)

ϕ = latitude of observer.

From the geocentric parameters δ and H the topocentric parameters δ_{corr} and H_{corr} are calculated by

$$A = \cos\delta\ \sin H \tag{C4}$$

$$B = \cos\delta\ \cos H - \rho'\cos\psi\ \sin\pi \tag{C5}$$

$$C = \sin\delta - \rho'\sin\psi\ \sin\pi \tag{C6}$$

$$H_{corr} = \arctan A/B \tag{C7}$$

$$\delta_{corr} = \arctan C/(A^2 + B^2)^{\frac{1}{2}} \tag{C8}$$

where ρ' is the geocentric distance normalised to the major axis and ψ is the geocentric latitude given by

$$\rho' = \frac{1}{297} \sin^2\phi \tag{C9}$$

$$\psi = \phi - 0.19 \sin 2\phi \ (\phi \text{ in degrees}) \tag{C10}$$

and π is the equatorial horizontal parallax of the centre of the moon.

Then the elevation E and azimuth A are calculated in the same way as in Appendix A, but by using δ_{corr} and H_{corr} instead of δ and H.

<u>HP 9100 B desk calculator program</u>

<u>User instructions</u>

Set:	DEGREES
Press:	END
Enter Program:	Side A followed by side B
Press:	END
Press:	CONTINUE
Display:	$\boxed{1 \quad 0 \quad 0}$
Enter:	ϕ (latitude in degs.) → y
	λ (longitude in degs.) → x (positive to the east)
Press:	CONTINUE
Display:	$\boxed{2 \quad 0 \quad 0}$
Enter:	H_γ (hours) → z
	H_γ (minutes) → y
	H_γ (seconds and fractional seconds) → x
Press:	CONTINUE
Display:	$\boxed{3 \quad 0 \quad 0}$
Enter:	α_o (hours) → z
	α_o (minutes) → y
	α_o (seconds and fractional seconds) → x

Press: CONTINUE

Display: $\boxed{4 \quad 0 \quad 0}$

Enter: $\Delta\alpha_o$ (seconds and fractional seconds) → x

Press: CONTINUE

Display: $\boxed{5 \quad 0 \quad 0}$

Enter: δ_o (degrees) → z

δ_o (min. of arc) → y

δ_o (seconds of arc and fract.) → x

Press: CONTINUE

Display: $\boxed{6 \quad 0 \quad 0}$

Enter $\Delta\delta_o$ (seconds of arc and fract.) → x

Press: CONTINUE

Display $\boxed{7 \quad 0 \quad 0}$

Enter: UT (hours) → z

UT (minutes) → y

UT (seconds and fract.) → x

Press: CONTINUE

Display: $\boxed{8 \quad 0 \quad 0}$

Enter: π (degs.) → x

(π = horizontal parallax interpolated to the nearest second of arc and converted to degrees)

Press: CONTINUE

Display:

Elevation	→	z
Azimuth	→	y
0 or 360	→	x

New time instant?

same hour	different hour	different day
Press: GOTO	Press: GOTO	Press: GOTO
+	+	+
4	1	0
a	3	7
CONTINUE	CONTINUE	CONTINUE

Title Elevation and azimuth of the moon

Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20		display		30	ACC+	60		enter		60	↑	27			
01	1	01	1	0	0	31	6	06	δ''	δ'	δ°	61	↓	25			
02	STOP	41		enter		32	0	00				62	x←	67	calculate α		
03	x→	23	λ	φ		33	÷	35				63	−	34	at Eph. Time		
04	d	17				34	Roll ↓	31				64	e	12	and store		
05	y→	40		store		35	+	33	convert to deg			65	×	36			
06	c	16				36	f	15	and store			66	x←	67			
07	CLEAR	20				37	Roll ↑	22				67	−	34			
08	2	02		display		38	÷	35				68	f	15			
09	STOP	41	2	0	0	39	÷	35				69	+	33			
0a	GOTO	44		enter		3a	↓	25				6a	GOTO	44			
0b	SUB	77	H_x^s	H_x^m	H_x^h	3b	+	33				6b	−	34			
0c	−	34				3c	y→	40				6c	0	00			
0d	c	16				3d	9	11				6d	0	00			
10	c	16	convert to deg			40	CLEAR	20		display		70	y→	40			
11	y→	40	and store			41	6	06	6	0	0	71	a	13			
12	7	07				42	STOP	41		enter		72	↓	25			
13	CLEAR	20				43	↑	27	$\Delta\delta'$			73	x←	67	calculate δ		
14	3	03		display		44	6	06				74	8	10	at Eph. Time		
15	STOP	41	3	0	0	45	0	00	convert to deg			75	×	36	and store		
16	GOTO	44		enter		46	÷	35	and store			76	x←	67			
17	SUB	77	α_0^s	α_0^m	α_0^h	47	÷	35				77	9	11			
18	−	34				48	y→	40				78	+	33			
19	c	16	convert to deg			49	8	10				79	y⇄	24			
1a	c	16	and store			4a	CLEAR	20		display		7a	b	14			
1b	y→	40				4b	7	07	7	0	0	7b	↑	27			
1c	−	34				4c	STOP	41		enter		7c	RCL	61			
1d	f	15				4d	ACC+	60	UT^s	UT^m	UT^h	7d	ACC−	63			
20	CLEAR	20				50	Roll ↑	22									
21	4	04		display		51	x→	23	store Universal								
22	STOP	41	4	0	0	52	b	14	Time,								
23	GOTO	44		enter		53	4	04	convert to								
24	SUB	77	Δα			54	3	03	Ephemeris Time								
25	−	34				55	+	33	calculate fraction								
26	c	16				56	6	06	of hour								
27	c	16	convert to deg			57	0	00									
28	y→	40	and store			58	÷	35									
29	−	34				59	↓	25									
2a	e	12				5a	+	33									
2b	CLEAR	20				5b	6	06									
2c	5	05		display		5c	0	00									
2d	STOP	41	5	0	0	5d	÷	35									

Storage

HEWLETT·PACKARD

Title: Azimuth and elevation of the moon

HEWLETT · PACKARD

Step	Key	Code	x	y	z	Step	Key	Code	x	y	z	Step	Key	Code	x	y	z
10	GOTO	44				40	×	36				70	↓	25			
11	SUB	77	calculate			41	ACC+	60				71	sin	70			
12	−	34	hour angle, H			42	2	02				72	↑	27			
13	c	16	and store			43	↑	27	calculate φ'			73	b	14			
14	c	16				44	c	16	and store			74	cos	73			
15	.	21				45	×	36				75	×	36			
16	9	11				46	Roll ↓	31				76	y→	40			
17	9	11				47	sin	70				77	f	15			
18	7	07				48	x⇄y	30				78	e	12			
19	2	02				49	.	21				79	TO POLAR	62			
1a	6	06				4a	1	01				7a	y⇄	24			
1b	9	11				4b	9	11				7b	a	13			
1c	5	05				4c	×	36				7c	↑	27			
1d	6	06				4d	↓	25				7d	RCL	61			
20	6	06				50	−	34				80	TO POLAR	62			
21	÷	35				51	y→	40				81	Roll ↑	22			
22	a	13				52	f	15				82	x⇄y	30			
23	−	34				53	↓	25				83	TO POLAR	62			
24	d	17				54	cos	73	calculate	H_{corr}		84	y→	40			
25	+	33				55	↑	27	and δ_{corr}			85	b	14			
26	x←	67				56	e	12	(parallax)			86	a	13			
27	7	07				57	×	36	and store			87	cos	73	calculate E		
28	+	33				58	b	14				88	x⇄y	30	and store		
29	y→	40				59	cos	73				89	cos	73			
2a	a	13				5a	↑	27				8a	×	36			
2b	CLEAR	20		display		5b	a	13				8b	c	16			
2c	8	10	ρ	δ	σ	5c	cos	73				8c	cos	73			
2d	STOP	41		enter		5d	×	36				8d	×	36			
30	sin	70	π			60	↓	25									
31	↑	27				61	x⇄y	30									
32	c	16				62	−	34									
33	sin	70	calculate			63	y⇄	24									
34	↑	27	e sin π			64	e	12									
35	×	36	and store			65	f	15									
36	2	02				66	sin	70									
37	9	11				67	×	36									
38	7	07				68	b	14									
39	÷	35				69	sin	70									
3a	1	01				6a	x⇄y	30									
3b	x⇄y	30				6b	−	34									
3c	−	34				6c	y⇄	24									
3d	↓	25				6d	a	13									

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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HEWLETT · PACKARD

Title Azimuth and elevation of the moon

Step	Key	Code	Display x	Display y	Display z
9 0	b	14			
- 1	sin	70			
2	↑	27			
3	c	16			
4	sin	70			
5	×	36			
6	↓	25			
7	+	33			
8	↓	25			
9	arc	72			
a	sin	70			
b	x →	23			
c	e	12			
d	c	16			
d 0	cos	73			
- 1	↑	27	calculate A		
2	b	14	and arrange		
3	sin	70	0 ≤ A < 360°		
4	×	36			
5	b	14			
6	cos	73			
7	↑	27			
8	c	16			
9	sin	70			
a	×	36			
b	a	13			
c	cos	73			
d	×	36			
4 0	↓	25			
(-) 1	−	34			
2	y →	40			
3	f	15			
4	a	13			
5	sin	70			
6	CHG SIGN	32			
7	↑	27			
8	b	14			
9	cos	73			
a	×	36			
b	f	15			
c	TO POLAR	62			
d	CLEAR X	37			

Step	Key	Code	Display x	Display y	Display z
c 0	IF x<y	52			
- 1	c	16			
2	7	07			
3	3	03			
4	6	06			
5	0	00			
6	+	33			
7	Roll ↓	31			
8	y⇄	24			
9	e	12			
a	Roll ↑	22		display	
b	STOP	41	δ	A	E
c	ACC +	60		Subroutine	
d	6	06			
d 0	0	00			
- 1	÷	35	convert		
2	Roll ↓	31	sec, min, hrs.		
3	+	33	into degs.		
4	f	15			
5	Roll ↑	22			
6	÷	35			
7	÷	35			
8	↓	25			
9	+	33			
a	1	01			
b	5	05			
c	×	36			
d	SUB/RET	77			
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

APPENDIX D

FORTRAN PROGRAM ANTOFF

<u>Input data</u>

	CC	Format	Description
n CARDS (one card for each measurement)	1 - 15	F 15.9	E1: Set elevation angle in degrees
	16 - 30	F 15.9	A1: Set azimuth angle in degrees
	31 - 45	F 15.9	E2: Elevation angle of the star at the instant of peak received power, in degrees.
	46 - 60	F 15.9	A2: Azimuth angle of the star at the instant of peak received power, in degrees
CARD n+1	70	R1	* (indicates end of data set)

Notes:
- Maximum number of measurements: 200
- Several data sets can be inserted simultaneously
- E2 = E1 and A2 = A1 simultaneously is not permitted. In this case calculate E_2 and A_2 for a time instant t_2 which is different from t_1 by a negligibly small amount, say one second.

<u>Output data</u>

The program reprints the input data and prints the resulting misalignments in elevation and azimuth with their associated standard deviations (see sample print-out below for SET-2).

```
      PROGRAM ANTOFF(INPUT,OUTPUT,TAPE1=INPUT)
C     THIS PROGRAM CALCULATES THE OFFSET OF THE BEAM CENTRE OF AN ANTEN-
C     NA RELATIVE TO THE INDICATED ELEVATION AND AZIMUTH ANGLE BY MEANS
C     OF STELLAR RADIO SOURCES,

      INTEGER  Y
      COMMON W, E, A, RAD
      DIMENSION  E(2,200), A(2,200),F(200),X(2),ACC(2),W(612)
     1           ,CA(200), CB(200),KM(200)

C     READ INPUT VALUES

    4 KN = 0
      I  = 0
      IK = 0
    5 I  = I+1
    6 READ 100,  E(1,I),A(1,I),E(2,I),A(2,I),Y
  100 FORMAT (4F15,9,9X,R1)
      IF (EOF(1)) 160, 10
   10 CONTINUE
      IF (Y,EQ,1R*) GOTO 20
      P = E(2,I) - E(1,I)
      Q = A(2,I) - A(1,I)
      IK = IK + 1
      IF ((P,NE,0,),AND,(Q,NE,0,)) GOTO 5
      KN = KN + 1
      KM(KN) = IK
      GOTO 6
   20 I = I-1

C     PRINT INPUT VALUES

      PRINT 110
  110 FORMAT (1H1,10X,*INPUT VALUES*,//,3X,*I*,7X,*E(1,I)*,9X,*A(1,I)*,
     1     9X,*E(2,I)*,9X,*A(2,I)*,/)
      DO 27  K=1,I
   27 PRINT 120,K,E(1,K),A(1,K),E(2,K),A(2,K)
  120 FORMAT (1H ,I3,2X,4F15,9)
      IF (KN,EQ,0)  GOTO 30
      DO 28   K = 1, KN
   28 PRINT 122, KM(K)
  122 FORMAT (1H0,8X,*INPUT DATA CARD NUMBER*,I3,*  IS SKIPPED BECAUSE*,
     1       * THE DIFFERENCES IN BOTH ELEVATION AND AZIMUTH ARE ZERO*)
   30 PRINT 121
  121 FORMAT (1H1,10X,*OUTPUT*,///)

      RAD = 3,1415926535898 / 180,

      X(1)=0,
      X(2)=0,
      ACC(1)=0,0001
      ACC(2)=0,0001
```

```
      CALL SSQMIN (I,2,F,X,ACC,100,,U,1000)
      DE=X(1)
      DA=X(2)

C     CALCULATE STANDARD DEVIATION OF THE OBSERVATIONS

      VV=0.
      DO 130  L=1,I
      VV=VV+F(L)*F(L)
  130 CONTINUE
      SIG=SQRT(VV/(I-2))

C     CALCULATE STD, DEVIATION OF THE MEAN OF THE VARIABLES

      ASUM=0,
      BSUM=0,
      ABSUM=0,
      DO 140  M=1,I
      P=E(2,M)-E(1,M)
      Q=(A(2,M)-A(1,M))*COS(E(1,M)*RAD)
      CA(M)=P/SQRT(P*P+Q*Q)
      CB(M)=(-Q/SQRT(P*P+Q*Q))*COS(E(1,M)*RAD)
      ASUM=ASUM+CA(M)*CA(M)
      BSUM=BSUM+CB(M)*CB(M)
      ABSUM=ABSUM+CA(M)*CB(M)
  140 CONTINUE
      ANOM=ASUM*BSUM-ABSUM*ABSUM
      IF (ANOM,EQ,0,) 141,142
  141 SIGE=0,
      SIGB=0,
      GOTO 143
  142 SIGE=SIG*SQRT(BSUM/ANOM)
      SIGA=SIG*SQRT(ASUM/ANOM)
  143 CONTINUE

C     PRINT RESULTS

      PRINT 150, DE,SIGE,DA,SIGA
  150 FORMAT (1H ,/////,5X,*MISALIGNMENT IN ELEVATION =*,F8,3,* DEGREES*
     1          ,5X,*ERROR =*,F6,3,* DEGREES*,//,
     2          5X,*MISALIGNMENT IN AZIMUTH   =*,F8,3,* DEGREES*,
     3          5X,*ERROR =*,F6,3,* DEGREES* )

      GOTO 4

  160 CONTINUE

      STOP
      END
```

```
      SUBROUTINE CALFUN (I,N,F,X)
      COMMON W, E, A, RAD
      DIMENSION F(1), X(1), E(2,200), A(2,200),W(612)
      DE=X(1)
      DA=X(2)
      DO 35  M = 1,I
      R=SQRT(DE*DE+(DA*COS(E(1,M)*RAD))**2)
      IF ((DA.EQ.0.).AND.(DE.EQ.0.)) GOTO 31
      GAMMA = ATAN2 (DA*COS(E(1,M)*RAD),DE)
   31 CONTINUE
      P = E(2,M) - E(1,M)
      Q = (A(2,M)-A(1,M))*COS(E(1,M)*RAD)
      DI = SQRT (P*P + Q*Q)
      BETA = ATAN2 (Q,P)
      IF ((DE.EQ.0.).AND.(DA.EQ.0.)) 33,34
   33 F(M) =-DI
      GOTO 35
   34 F(M) = R*COS(GAMMA-BETA) - DI
   35 CONTINUE
      RETURN
      END
```

```
SUBROUTINE SSQMIN(M,N,F,X,E,ESCALE,IPRINT,MAXFUN)
```

CDC 6400 LIBRARY ROUTINE E4 BKY

INPUT VALUES

I	E(1,I)	A(1,I)	E(2,I)	A(2,I)
1	16.202800000	88.793430000	16.335480000	88.812180000
2	18.601410000	91.623140000	18.616460000	91.642130000
3	20.880830000	94.494510000	20.893060000	94.510600000
4	28.172510000	332.792550000	28.156080000	332.811580000
5	27.520950000	334.033260000	27.484070000	334.031160000
6	26.236620000	336.573220000	26.179300000	336.607430000
7	25.312880000	338.509970000	25.307240000	338.522970000
8	24.503300000	340.471950000	24.493010000	340.498270000
9	23.540850000	343.123850000	23.533400000	343.146100000
10	22.908980000	345.137040000	22.903720000	345.155020000
11	59.036380000	304.199460000	59.059610000	304.186990000
12	56.506700000	305.368300000	56.522130000	305.356740000
13	54.007530000	306.711150000	54.034280000	306.695770000
14	51.561570000	308.200210000	51.559550000	308.201510000
15	49.167300000	309.815240000	49.185050000	309.802700000
16	45.685680000	312.442140000	45.683780000	312.443664000
17	43.446220000	314.314170000	43.455400000	314.306190000
18	51.486610000	234.722910000	51.488700000	234.717760000
19	50.209540000	237.743380000	50.198720000	237.768010000
20	48.890150000	240.638020000	48.887920000	240.642740000
21	47.533460000	243.416050000	47.528880000	243.425120000
22	46.144073000	246.086590000	46.141732400	246.090955000
23	40.338260000	255.865090000	40.325850000	255.884180000
24	38.841580000	258.122890000	38.821540000	258.152570000
25	37.332600000	260.319940000	37.307360000	260.356090000
26	59.735550000	172.182420000	59.732400000	172.113530000
27	59.907370000	181.430680000	59.907930000	181.361160000
28	59.806770000	186.055920000	59.808390000	186.009810000
29	59.583040000	190.641660000	59.586830000	190.580900000
30	57.035030000	145.820950000	57.022090000	145.761990000
31	58.226040000	151.891490000	58.213980000	151.822180000
32	59.195860000	158.289500000	59.185430000	158.209480000
33	54.986370000	146.549080000	54.977890000	146.511070000
34	57.109960000	158.548320000	57.105270000	158.513460000
35	16.326360000	285.018410000	16.306520000	285.043220000
36	14.110100000	287.807760000	14.105210000	287.813960000

OUTPUT

MISALIGNMENT IN ELEVATION = -.001 DEGREES   ERROR = .010 DEGREES

MISALIGNMENT IN AZIMUTH = .009 DEGREES   ERROR = .012 DEGREES

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SHAPE Technical Centre, The Hague, The Netherlands

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by A. Wallrabe, March 1974, v + 61 pp. incl. illust.
(Technical Memorandum STC TM-413)

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The accuracy of the method is assessed both, by estimating the errors associated with the various measurement parameters, and by determining the scatter of the measured results.

The method has been applied to the experimental ground terminal SET-2 at the SHAPE Technical Centre. Within the accuracy limits of 0.01° for each axis no misalignments could be detected.

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