Canadian Centre for Experimental Radio Astronomy
http://www.ccera.ca

Memo: 15 *Measurement of Mean Lunar Disc Temperatures at 2.6 cm Wavelength.*

To: Interested parties

Cc: Open Source Radio Telescopes

From: Ken Tapping, ken.tapping@shaw.ca

Date: Oct 8, 2022

Subject: *Measuring the Moon's Temperature at 2.6 cm Wavelength*

This memo describes the design and implementation of a radio telescope system to measure the temperature of lunar soils at a wavelength of 2.6 cm. The observation methods, data and analysis, together with some results are also discussed. This project is not front-line science. Its objective was to see if usable information about the Moon could be obtained through a backyard experiment.

Introduction

The Moon is a strong source of radio waves at centimetre wavelengths. This has been known for a long time. For example, lunar radio emissions are discussed in detail by Pawsey and Bracewell (1955) and Kraus (1966). For background information for users of small radio telescopes, the older references are usually most useful. Although these emissions are strong, the Moon is not often a subject of interest to users of small radio telescopes. this is probably due to for most its history, backyard, small instrument radio astronomy has been done at metre and decimetre wavelengths. It is only recently that low-cost but highly-sensitive receiver front ends have become available for C-Band (2-4 GHz), Ku-Band (12.4-18 GHz) and Ka-Band (18-24 GHz). However, for the two higher-frequency bands, most of those front ends have integrated feeds and do not permit any easy and consistent method of calibration. It is possible to detect the Moon using these components, but making consistent measurements over several or more lunar phase cycles requires a comparison or calibration source to be included in the receiver.

This situation changed when one dealer in satellite broadcasting and communication components advertised low-noise Ku-Band low noise block down-converters with standard waveguide (WR75) inputs for $25 each. (Figure 1). These devices usually retail for hundreds of dollars. A waveguide input permitted the attachment of a waveguide directional coupler for injection of calibration noise, which is exactly what is needed for a long-term lunar temperature measurement project.

Figure 1: The Ku-Band block downconverter used for this project. An older style amplification device is included for comparison purposes.

Lunar Radio Emissions

For some odd reason, a number of amateur radio publications state that lunar radio emissions are reflected solar radio emissions. This not the case; the Moon is a cosmic radio source in its own right. The Moon does reflect solar radio emissions, but these reflections are very weak.

Lunar radio emissions are thermal, due to the Moon having a temperature. They are the emission due to a black body with a mean temperature of around 225K. Like the Earth, the Moon receives its heat from the Sun. At our distance from the Sun, we receive about 1600 Watts per square metre. The Moon has no atmosphere, so all this energy gets to the surface. Roughly 12% of that energy is reflected back to space. The rest warms the surface layers. With no atmosphere or greenhouse effect to inhibit the radiation of heat to space during the lunar night, the lunar daily temperature changes in the surface layers are huge, ranging from above 100 C during the day to below -150 degrees C during the night.

The figure below shows the mean flux density of lunar thermal emissions over the wavelength range 1-100 cm. At wavelengths shorter than about 10cm, the Moon should be easily detectable with back-yard sized dishes.

Figure 3: The approximate radio emission spectrum of the Moon. The line shows the flux density of the lunar emission as a function of wavelength. A Jansky is 10^{-26}.w.m^{-2}.Hz^{-1}.

The periodic, lunar-daily (although non-sinusoidal) temperature changes at the surface propagate down into the ground as "heat waves". As the depth increases the amplitude of the temperature variation decreases, until a depth is reached where the temperature is unchanging. Since lunar material takes time to warm up and cool, there is a phase delay between the temperature variations at the surface and those below the surface which increases with depth.

At infra-red wavelengths, the emissions come from the surface, and vary dramatically with the ground temperature during the lunar day. As the observing wavelength increases, we see deeper and deeper into the lunar surface layers. The deeper we go, we expect the temperature variations to decrease and the delay compared with the phase of the Moon to increase. Observations at centimetre wavelengths allow us to stick our fingers into the lunar soil without the travel expenses.

Objectives of the Experiment

The science objective of the experiment is to monitor the changes in temperature at the depth below the lunar surface where most of the 2.6 cm emission originates, and to formulate a simple model to help understand what is going on. This leads to the other objective, to develop the hardware and observing methods needed to do this.

Measurements of the absolute mean temperature of the lunar disc are beyond the capabilities of the equipment used here. Since there are no sources in the sky with which the intensity of the lunar emissions can be compared, making such measurements would require absolute information of the effective collecting area of the antenna and the value of the calibration signal being leaked into the signal line. Therefore, this project is based upon relative change in the lunar emissions compared with the calibration signal, the absolute value of which is not known, but is assumed to be constant over time.

Daily lunar measurements were made over at least several lunar cycles in order to see the real variations through equipment inconsistences and fluctuations in tropospheric attenuation. The mean value in the variations in the Moon/Cal ratio are assumed equal to the mean lunar temperature, and the excursions in the ratio to be linearly related to deviations from that mean temperature.

These measurements are then compared with a simple heat transfer model.

Designing the Experiment

At a wavelength of 2.6 cm, and at shorter wavelengths, the troposphere is a problem; clouds and rain attenuate the cosmic radio emissions and contribute noise of their own, which vary with what is in the sky and how it is moving. One method to get rid of this tropospheric noise and baseline fluctuations that is used on large antennas is to have an auxiliary feed (a "reference" feed), offset slightly from the "main feed". For large dishes, with long focal lengths, the main and reference beams overlap and diverge so slowly they finally separate many kilometres from the antenna. Both beams see more or less the same troposphere, but only one sees the source, so subtracting the reference beam signal from the main beam signal largely removes the tropospheric noise. For small dishes, such as the one used in this project, the short focal length means the reference beam ceases to overlap the main beam just a few metres from the antenna. The two beams "see" different patches of sky, each with its own varying noise output. The result is the subtraction actually makes the situation worse, Consequently, when a single-antenna radiometer is used, the best approach is to use a single feed and record everything it receives, and try to work around the problems associated with single-antenna, total power radiometry.

As viewed from the Earth, the Moon has a diameter of about 30 arc-minutes, and it has a mean temperature of about 225 K. To measure the integrated emission from the Moon, getting the average disc temperature, one would ideally have to use an antenna which "sees" the whole lunar disc with equal sensitivity. This indicates a small antenna with a large beamwidth should be used. However, this raises some serious problems.

With a single antenna, there are two ways to make a measurement; we either wag the antenna on and off-source, noting the difference in noise level, or we put the antenna at a fixed position on the sky and let the rotation of the Earth carry the source through the beam. Both methods have their disadvantages.

Any antenna with a beamwidth large enough to see the lunar disc with more or less uniform sensitivity is going to receive a lot of ground noise. Moving the antenna a few beamwidths off source is going to lead to the antenna seeing a different patch of ground, so part of the change in level as the antenna is moved on and off-source is going to be due to changing ground radiation. Unless this component can be separated from the contribution due to the Moon, the accuracy of any Moon temperature measurements will be degraded.

The alternative approach, letting the Moon drift through the antenna beam also has disadvantages. To see the Moon with uniform sensitivity using a dish with a single feed will require an antenna with a beamwidth of at least around four degrees. This means, to get a transit plus some baseline before and after will require at least an hour. During that time the receiver gain will drift, and different patches of tropospheric material will drift through the antenna beam, causing the baseline to change, again making it hard to make an accurate measurement. Noise fluctuations from the troposphere can change the antenna temperature by several Kelvins, and with a small antenna, the lunar noise contribution will be comparable. Accurate measurements will again be hard to make. The result is that in order make useful measurements, some compromise of the ideals of the experiment is needed, namely using an overlarge dish.

A 1.2m dish will have a beamwidth in the region of 1.5 degrees. When pointed at the centre of the lunar disc, the sensitivity of the antenna to emissions from the lunar limb will be around 10% lower. On the other hand, the antenna temperature increase due to the Moon will be around 20-30 K, which is comfortably larger than typical tropospheric noise fluctuations. and a half-hour observation should be adequate for recording a transit plus some baseline before and after. Ground radiation should be less of a problem. Therefore, a 1.2 m dish was used for the experiment, on an elevation-only mount intended to catch the Moon around meridian transit, when it is at its highest. The Moon is a strong radio source at centimetre wavelengths, so a wide receiver bandwidth is not needed, and a narrow bandwidth reduces the chances of interference by manmade signals. In addition it makes it possible to use software defined radio devices for the receiver backend.

The Hardware

Figure 3: The radiometer used in this experiment.

A 1.2 m diameter dish is mounted so it is looking towards the south and can be adjusted in elevation. The receiver front-end and calibration injection are located in the focus package, which is enclosed in a piece of 15cm diameter PVC pipe. The rest of the electronics is in the equipment box, which is mounted on the frame behind the dish. The antenna elevation drive uses an actuator from an old satellite TV antenna.

A block diagram of the system is shown below.

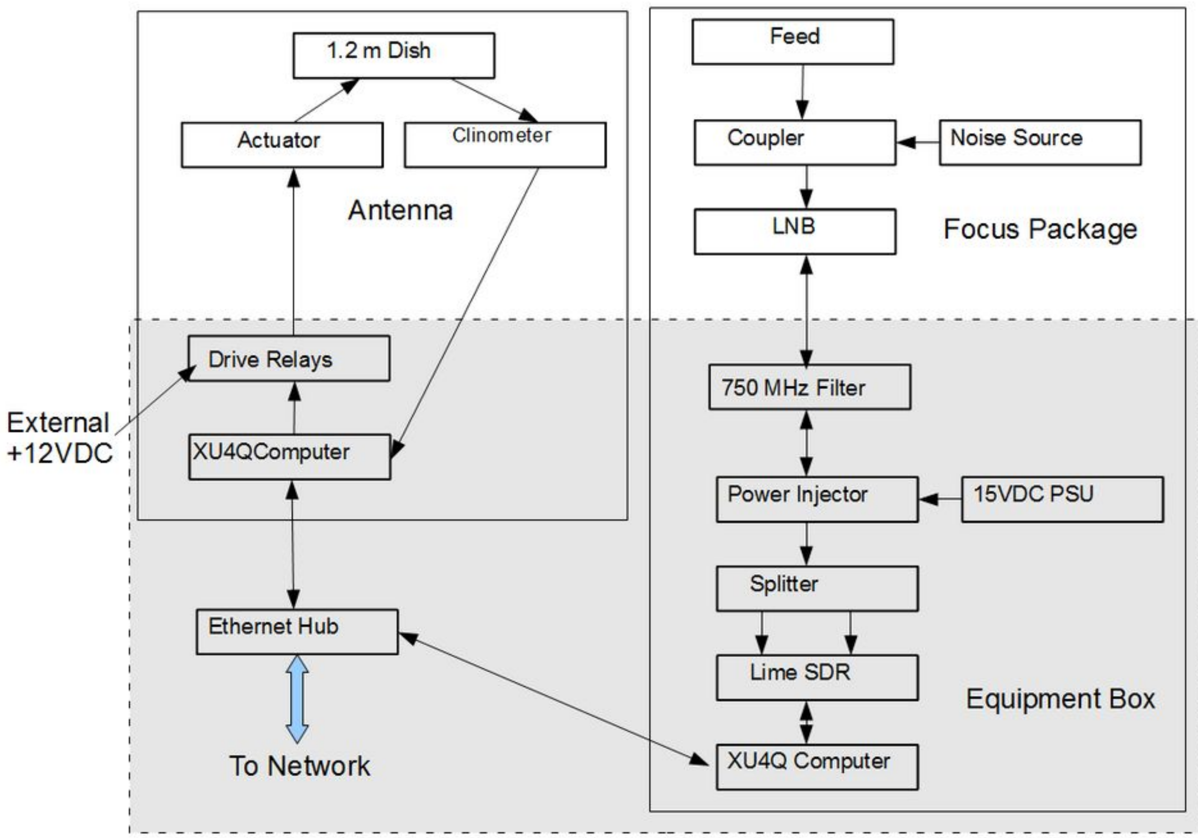

Figure 4: System Block Diagram. Components in the shaded grey box are located in the equipment box

The Antenna

Before deciding to use a 1.2m dish for the project, some trials were conducted using a 60cm dish, which would actually be more appropriate for the experiment. At 2.6 cm wavelength, such a dish would have a half-power beamwidth of around three degrees. Since the mean lunar disc temperature is around 225 K, this would give an antenna temperature increase of about:

$$T_A \approx 225\left(\frac{0.5}{3}\right)^2 \approx 6\,K$$

For modern receiver components, this would be a very easy temperature increase to measure. However, getting usable measurements is not so much a matter of signal to receiver noise ratio as one of signal to tropospheric noise ratio. With tropospheric noise variations ranging from nearly zero to a few Kelvins, most of the time lunar observations using a 2ft dish simply tdo not stand out well enough above the baseline variations due to the tropospheric noise variations to be measurable with sufficient accuracy. So a 1.2m dish was used, with a beamwidth of around 1.5 degrees, which yielded antenna temperature increases when pointed at the Moon of roughly 25K.

In this "drift transit" measurement method, the elevation of the antenna is set in advance and the Moon drifts through the antenna beam. In principle the antenna elevation could be set manually for each observation. However, the Moon can transit at extremely inconvenient times as it moves around its orbit, so the antenna elevation would often have to be set hours in advance. For this experiment it was not possible to dig holes and make a secure foundation for the antenna mount; it would have to sit on the ground. Therefore, in addition to flexure in the mount, heaving of the ground as it freezes and thaws, gets wet and dries can all move the antenna. Therefore an elevation sensor is fitted, which uses a gravity reference rather than indicating the angle between the antenna and the mount, and automatic telescope control was added to keep the antenna elevation constant with respect to the gravity vector. The antenna elevation is checked automatically every ten minutes of so, and corrected as needed.

The Receiver

One problem encountered in the construction was that I only have a few bits of WR75, which is the waveguide size used in the input to the LNB. However, I have quite a lot of WR90 waveguide. This is designed to operate over the band 8 - 12.4 GHz. Fortunately the intended observing frequency for the experiment was 11.5 GHz. This falls well in the operating range for WR90 waveguide, so the big question was how to fix a the short section of WR75 attached to the LNB to the WR90 waveguide used for the feed and calibration insertion. Adaptors, even surplus ones, are hard to find and expensive. Therefore, before looking at buying an adaptor, a crude adaptor was made by simply soldering a short section of WR75 to a short section of WR90, carefully lined up, flange to flange. This has proved perfectly adequate for this experiment, so that is what was used.

The local oscillator in the LNB is 10.75 GHz, and the chosen IF 750 MHz. This is below the stated IF band for the device, but LNB amplifiers are usually broad-band and moving outside the main operating band was assumed to get away from the bands most heavily occupied by satellites. The observing frequency is 11.5 GHz. For such a strong source, a large IF bandwidth is not needed; 4.5 MHz is used here, along with a 5s integration time.

The IF signal (with DC for the LNB running up the centre conductor) is taken down one of the feedlegs on the dish to the equipment box, where it is split. This split is not really necessary, but the Lime SDR has two signal inputs, and it seemed a shame to abandon a signal channel, although in principle the two data streams should be identical.

The Lime SDR is controlled by and sends data to an Odroid XU4Q single-board computer. The data are logged on a local memory stick, which is used as a cheap solid state disc. Periodically the data files are pulled off that disc, over the network, to a desktop computer that is used for data analyis.

The Software

Marcus Leech has produced a complete software package which can handle most common SDR devices. It contains elements of GNU Radio, and can handle total power, differential, spectral and correlation radiometers. The observations are set up using a simple shell script. The software comes on a chip along with Linux and a bunch of useful tools for reading antenna positions and driving antennas. Installing it on a new XU4Q just involves copying the micro-SD chip and changing the IP address.

Setting Up

Calibration Setup

The assumed value for the Moon's mean temperature is used to establish the calibration of the system. The setup of the calibration system is just to get the calibration deflection in the range of lunar deflections. This was done by pointing the antenna at the zenith (assuming an antenna temperature of 10K) and then putting absorbing foam in front of the feed (assuming 290 K). The deflection change between sky and load was then compared with the calibration deflection:

$$T_{cal} = (T_{load} - T_{sky}) \left(\frac{\text{Cal Deflection}}{\text{Sky Deflection}} \right)$$

Although the absolute value of the calibration signal is not used, it is useful to get the calibration deflection more or less equal to the average lunar deflection. This requires having a rough idea of the amount of calibration noise getting into the signal line and attenuating it as necessary.

Elevation Setup

The system uses a fixed azimuth antenna which is adjustable in elevation only. It was pointed more or less at the southern meridian, and the elevation checked by scanning up and down as the Sun transited. Close to the meridian, the elevation of the source is almost constant, and any small azimuth error would just change the source transit time. Unlike the Moon, the elevation of the Sun is easily calculated. The digital SDR device was unusable for this because of the time taken for the data to get to the output. In a transit instrument the scanning has to be a lot faster than the rate at which the Sun is moving through the beam, so the SDR device was replaced by an IF amplifier, a diode detector and a meter. As the Sun transited, the signal was peaked manually and the elevation sensor value noted. The difference between the indicated and actual elevation of the antenna is used manually in setting up the antenna rather than building into the control software because the time saving in setup would be marginal and if the correction changed, there would be no need to stop the system or intervene in the software.

Measurement Method

At any moment, with the antenna pointed in a particular direction, the noise input to the receiver system consists of the noise generated in the receiver's input circuits, and the noise coming from the antenna. The latter comprises noise from the ground, the atmosphere and the radio source being observed:

$$T_{input}=T_r+T_{ground}+(1-\alpha_0)T_{trop}+\alpha T_{moon}\cdot\left(\frac{\omega}{\Omega}\right)$$

In the above T_r is the receiver noise temperature, T_{ground} is the thermal noise contribution to receiver input from all the ground "seen" by the antenna. The parameter α_0 is the transmissivity of the troposphere averaged over the antenna beam; α is the tropospheric transmissivity averaged over the patch of sky covering the lunar disc. The lower and upper-case omegas are respectively the solid angles subtended by the Moon and the antenna beam. The mean tropospheric transmissivity over the lunar disc is not necessarily the same as the mean transmissivity over the antenna beam. The ground noise and tropospheric noise contributions are functions of the direction in which the antenna is pointed. Therefore, the approach used here is to keep the antenna stationary, which will keep the ground contribution more or less constant, apart from slow variations due to changing ground temperature or wetness during the day.

The Moon takes about 20 minutes to drift into and completely out of the antenna beam. The total power stability of the receiver is unlikely to change significantly over that time. However, in the 25 hours or so between transits the gain may change significantly, especially when high accuracy and consistency between measurements are needed. For this reason a solid-state noise source is turned on for a minute or two every hour or so for all the time the radiometer is running.

The Observations

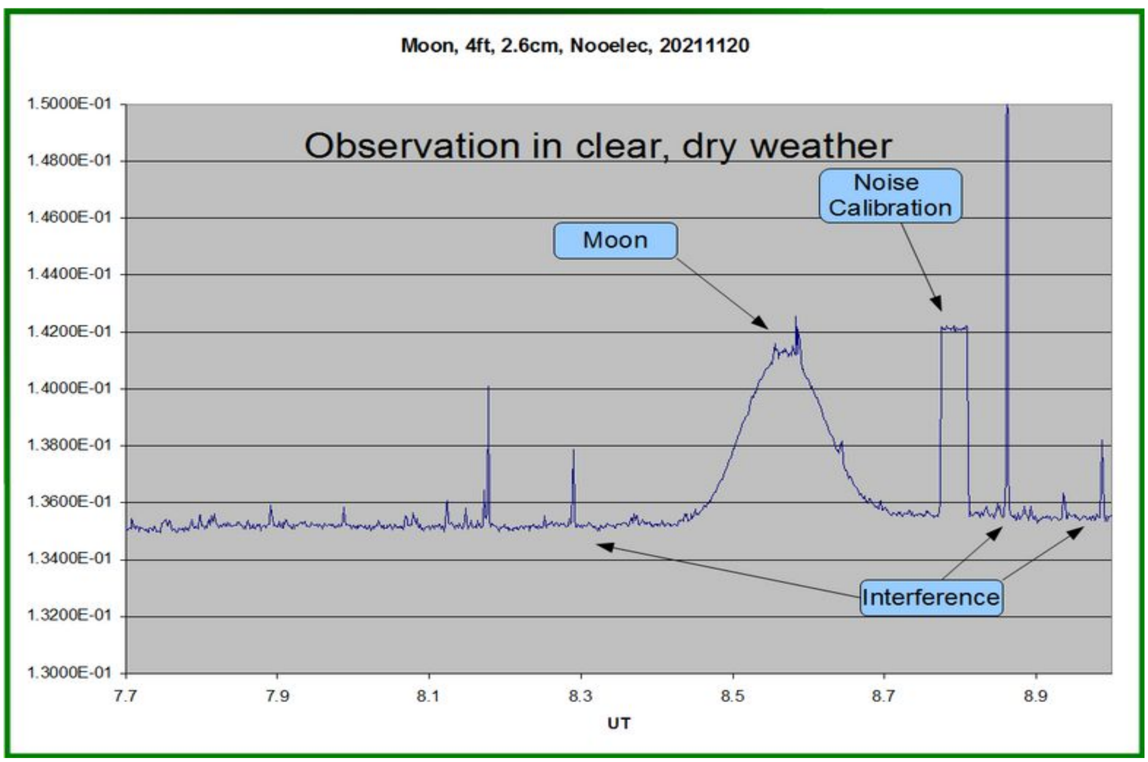

Figure 5: An example of a lunar transit as recorded with the 1.2m dish.

The antenna is set to the elevation of the Moon and the source allowed to drift through. This observation is a nice one for analysis, apart from the interference spikes at the peak.

However, observations at this wavelength are strongly affected by the weather, which can render the observations unusable, as below. The lunar peak, just before 0800 UT is narrower because the *x*-axis is more compressed in this plot. Structures in the troposphere can make quasiperiodic wriggles in the baseline as they drift past. The spikes are interference.

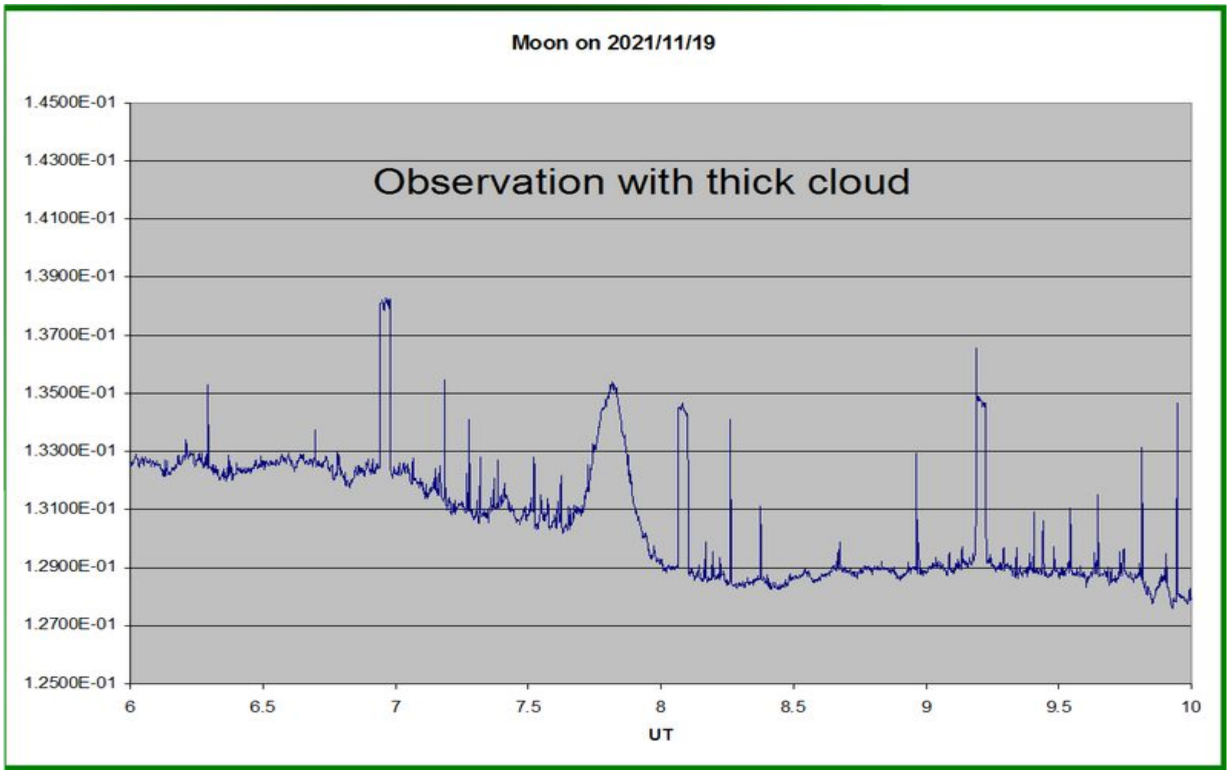

Figure 6: Lunar transit through thick cloud. This observation is unusable.

Sometimes the weather can completely obliterate the observation. Heavy rain has significant attenuation at 2.6 cm wavelength. This has two effects: firstly it weakens the desired signal, which has to find its way through it, and secondly, the lossy rain cloud generates thermal radio noise of its own, as does any attenuator. The big bump between 0300 and 1200 UT is thermal noise from the rain clouds. The little spikes are the calibration signals. The lunar signal is totally obliterated. There is a bit of gain compression between the calibration spikes riding on top of the rain and those on the baseline, where it was not raining. There is no sign whatsoever of the Moon.

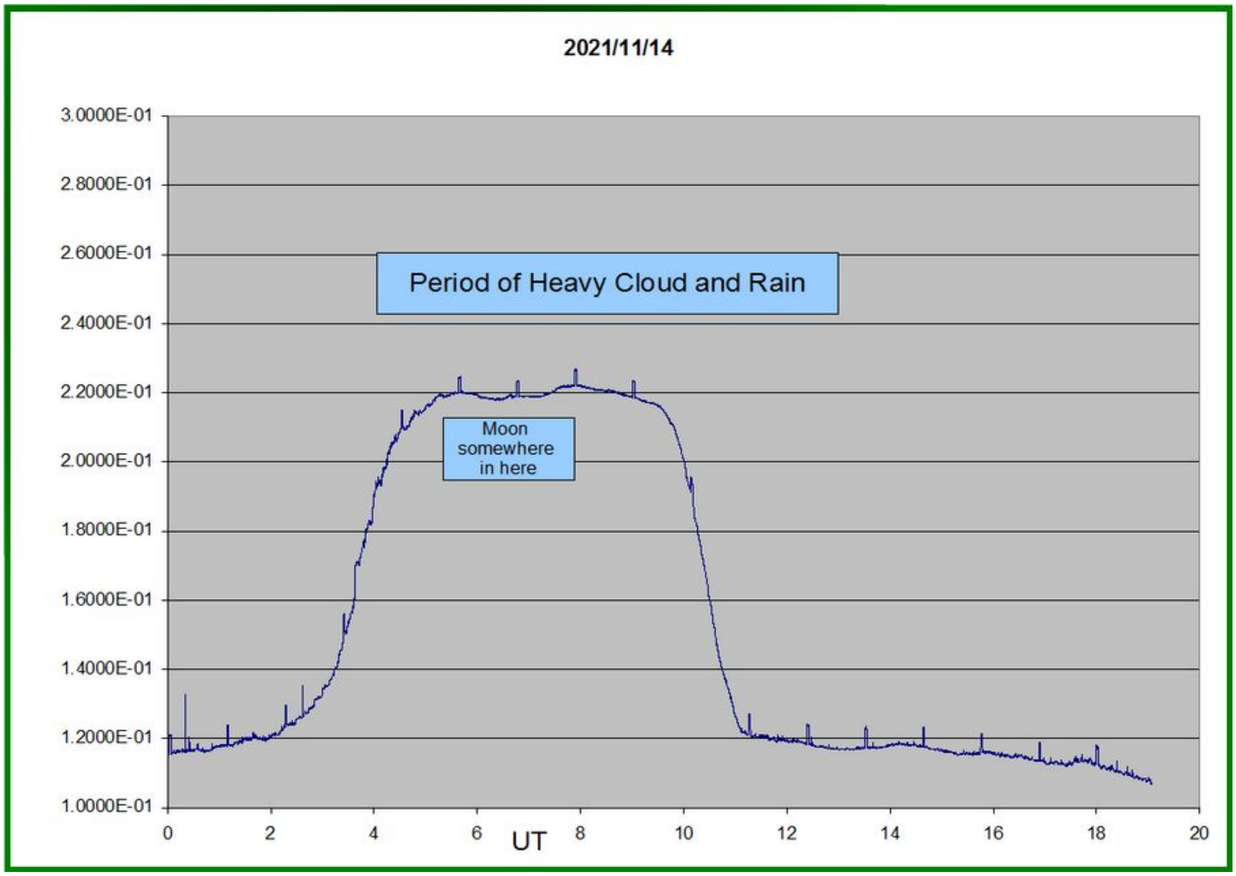

Figure 7: Received signal over a day with a rainstorm. During the event there is strong radio emission from the weather, but nothing visible from the Moon.

Maybe an instrument of this kind would be a useful tool in the new science of "radio meteorology".

Analysis

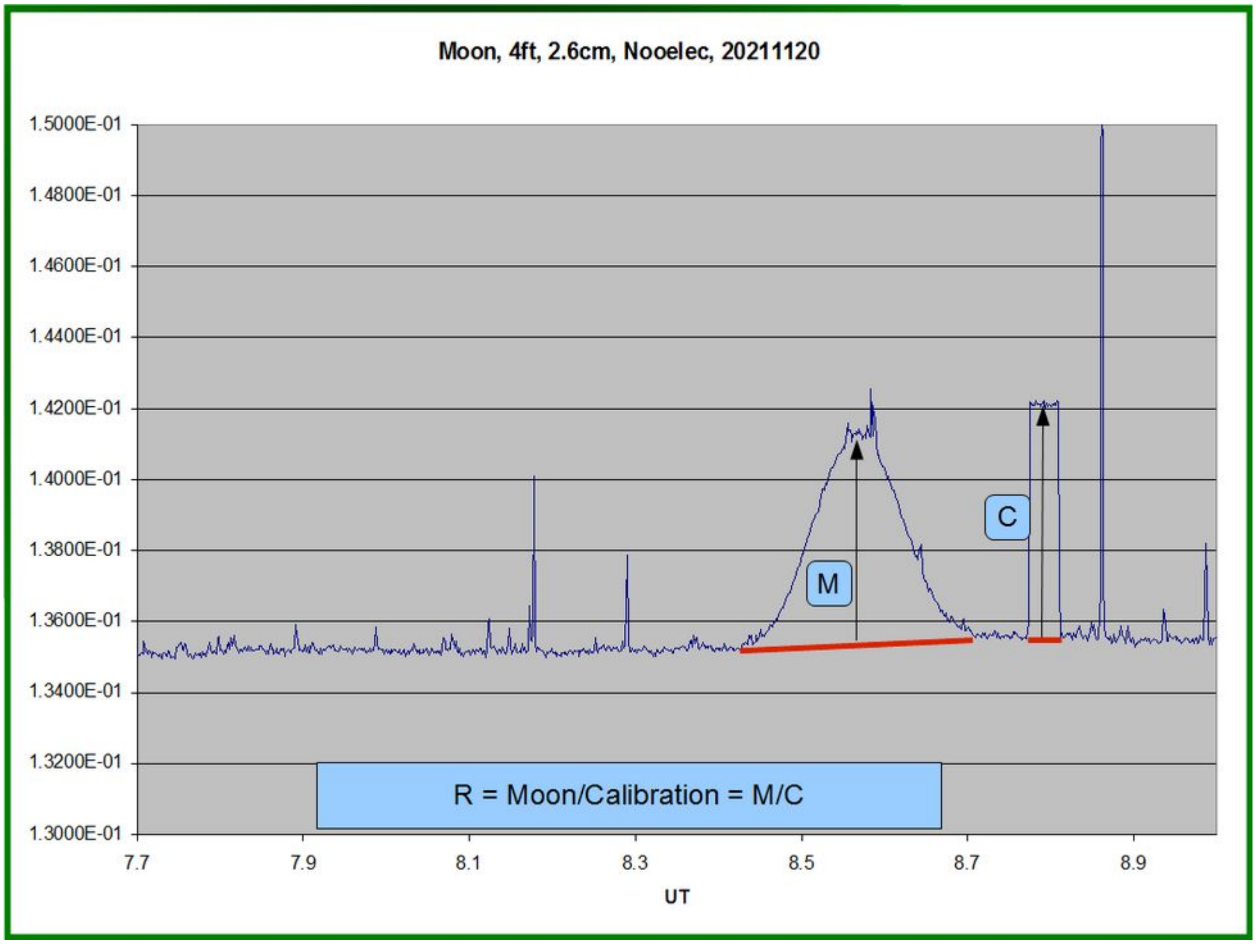

Figure 8: Method of measurement. Note the baseline change during the lunar transit. The implicit assumption is that the change during the transit was linear. This is a possible cause of error.

With only one measurement a day, processing the data manually does not take much time, and it is easy to deal with instances where interference spikes distort the transit, or a calibration spike is superimposed on it. This sort of observation would not be easy to automate in a precise way. A problem with this measurement is that the baseline changed during the lunar transit. In the analysis it is assumed to have happened linearly during the transit. However, the step to the right of the peak might have been the end of the interference or some sort of gain step. It is issues like this that cause a lot of frustrating scatter in the data.

Raw Measurements

The Lime SDR has two inputs. The IF signal from the front-end LNB is split between them. In principle the results should be identical, but little differences in the signal processing, together with inevitable errors in manual attempts to make precise measurements from data with baseline slopes, spikes and level changes lead to manual measurements using the results from the two data channels being different. In the plot below, one channel is marked with yellow points, the other with blue, and the mean values wirh red points. The change in emission over the lunar cycle is quite obvious,

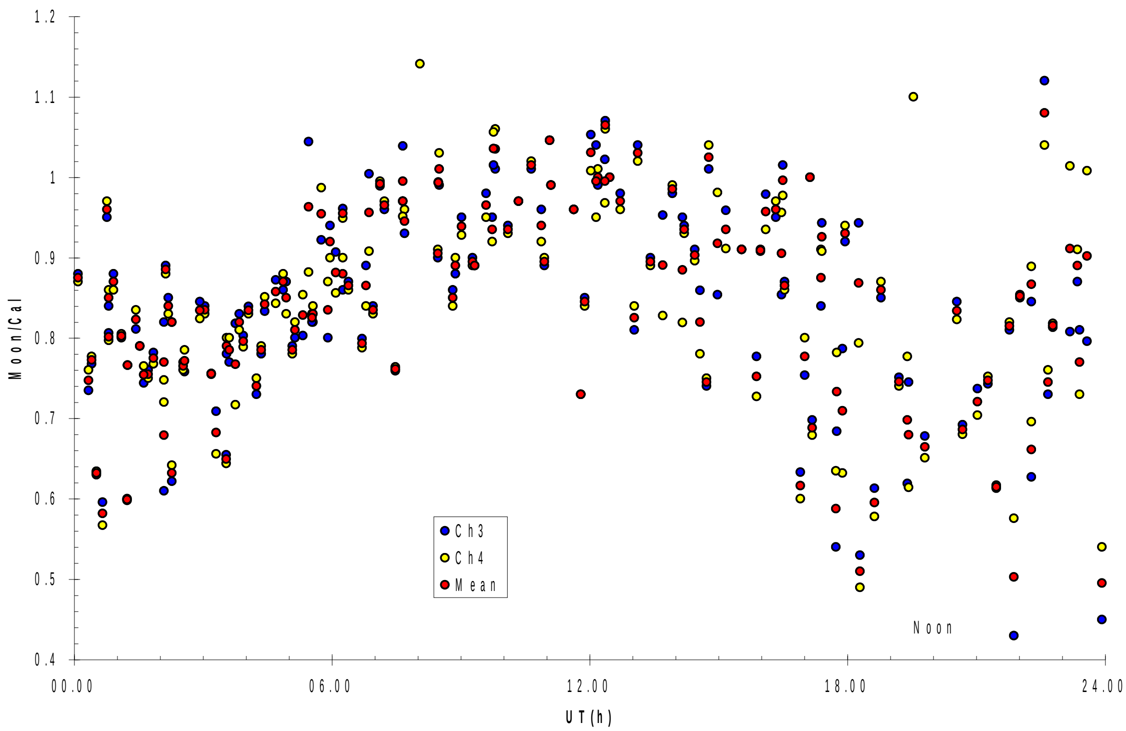

Figure 9: Raw measurments from the two data channels (yellow and blue), together with the mean value (red). Isolated red dots indicate three identical values.

The horizontal scale is UT. For measurements made with the antenna pointing south, the New Moon always transits close to the time of solar transit (nominally 20:00 UT at 120 degrees W Longitude) and the Full Moon at local midnight (08:00 UT). The great increase in noisiness arount 20:00 UT, when the Moon is New, is due to the close proximity of the Sun, which at 2.6 cm wavelength is around two orders of magnitude brighter than the Moon. The Sun in the sidelobes distorts the baseline and makes the amplitude of the lunar transit hard to determine. Since the New Moon is always close to the Sun this issue is unavoidable. However, this plot shows a clear change in the strength of the lunar signal as a function of phase.

It is easy to see if the dataplot (means only to reduce confusion) is repeated over three days, providing a view of three cycles. A fitted sinewave is added. In this case the *x*-axis is lunar phase angle, with 0 degrees corresponding to New Moon and 180 degrees the time of Full Moon.

The sine wave is not a very good fit, significantly underestimating the temperature around New Moon, but it is clear that at 2.6 cm wavelength, the maximum temperature occurs around 60 hours after the Full Moon, about 2.5 days. A test of the model would be that the depth of lunar soil producing a peak emission that long after the Full Moon should produce the observed modulation depth.

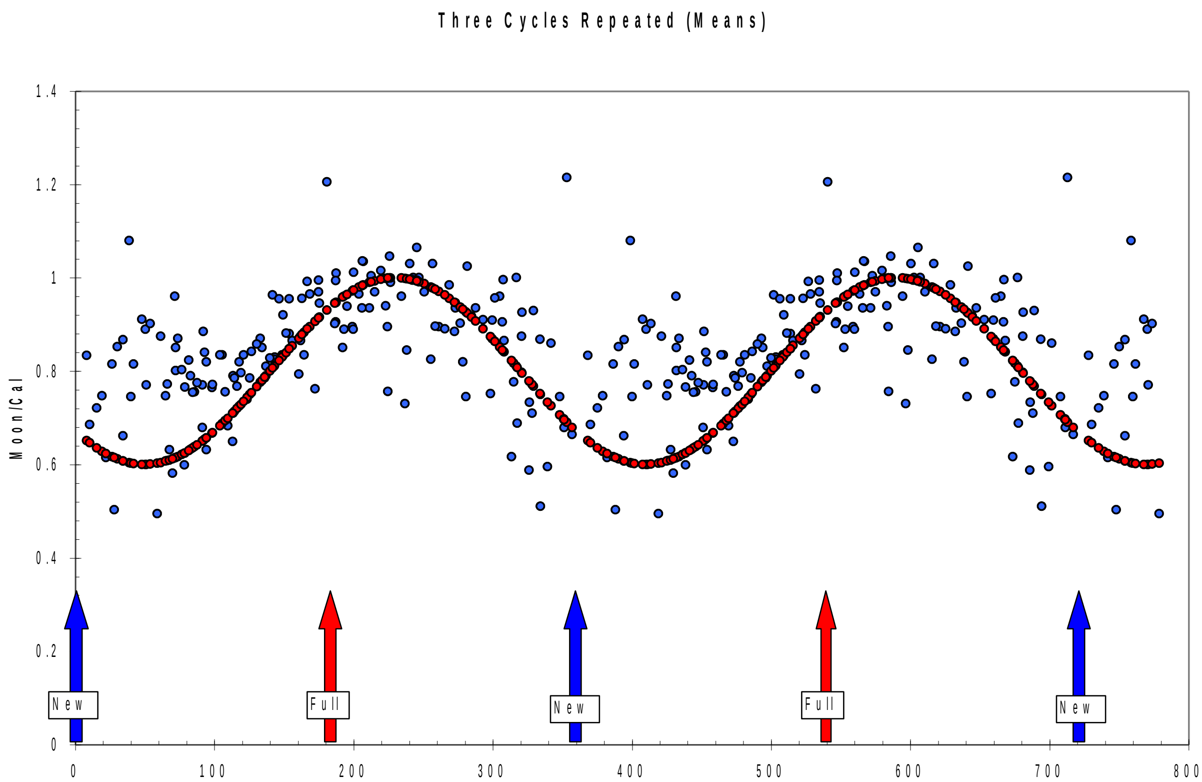

Figure 10: Mean Daily values repeated to show the variation more clearly. The fitted sinusoideis not a model , just a tool for identifying the day of peak radio brightness temperature, which is about 2.5 days after Full Moon. Horizontal axis is UT hours.

The "Model"

When the Sun shines on the lunar surface, the immediate surface layer starts to heat up, and then, when the lunar day ends, it cools off again, and during the lunar night, it can get very cold. Heat transfers downward by conduction, and during the night, the relatively warm deeper layers send heat back up to the surface, where it radiates off into space. Thermal conduction takes time, so the diurnal temperature variation at greater depths increasingly lag the temperature changes at the surface, and are smaller. There is a depth where the temperature does not change. Data obtained during lunar visits suggest that below a depth of about a metre, the temperature is constant. Two versions of the model are examined here. One is a calculation of temperature variations at a single location at the centre of the lunar disc, on the equator, and the other is an attempt to more realistically describe the situation.

When we look (for example), at the Full Moon, it is sunrise on one limb of the Moon and sunset on the other. On the sunrise side the ground is just starting to warm after the cold, lunar night. On the sunset side the ground is starting to cool after a hot, lunar day. In the middle of the disc it is noon. Therefore, a calculation of the mean disc brightness temperature requires calculations for each point on the lunar disc, taking into account the lunar time of day at each location.

The approach here is to use measured data for the thermal conductivity and thermal capacity of lunar basalt, and to adjust the radio absorption coefficient until the radio brightness temperature variation has the observed amplitude and phase delay.

Modelling Considerations

The Moon is basically a ball of volcanic rock - mainly basalt. Over billions of years, impacts by large and microscopic cosmic particles, together with degradation of outcrop surfaces by lare temperature variation, the surface is covered by a fine dust of lunar material. This is mixed with larger chunks. This lunar soil, or regolith was sampled during the Apollo missions and extensively studied. For example, see Cadogan (1981) and Faure and Mensing (2007). This layer is quite thick (> metres in some places), and at 2.6 cm wavelength, all the optical thickness for the emission almost certainly is located in the regolith layer; we are measuring the temperature of the regolith. The thermal qualities of finely-divided basalt dust in a vacuum are not clear. However, since the calculation is being done for emissions at 2.6 cm wavelength, which is far larger than the grain sizes (about half the weight of a lunar soil is made up of grains less than 60 to 80 microns in size) or the spaces between them, the regolith is treated as solid basalt.

The Temperature and Heat Flow Model

Parameter	Value	Units
Solar Heat Input	1365	$W.m^{-2}$
Lunar Albedo	0.12	
Area of Element	1	m^2
Thickness of Element	0.01	m
Number of Element Layers	100	
Thermal Capacity*	$3.2x10^6$	$J.m^{-3}.K^{-1}$
Thermal Conductivity*	1.5	$W.K^{-1}.m^{-1}$

* https://www.engineeringtoolbox.com/specific-heat-solids-d_154.html, and

Halbert D., Parnell J.: "Thermal Conductivity of Basalt Between 225 and 290 K", Meteoritics and Planetary Science, Wiley Online Library, https://onlinelibrary.wiley.com/doi/full/10.1111/maps.13829 (2022).

See also Clegg *et al* (1966).

The model consists of 100 layers of basalt regolith stacked on top on one another, each 1 cm thick, located at the centre of the lunar disc. The substrate is at a constant temperature and the surface of the top layer is receiving sunlight, varying in intensity of the lunar day, and losing heat by radiation to cold space. Heat propagates up and down the stack of layers. Basically the problem resembles a string of resistors with capacitors to ground at each junction, with a periodically varying voltage on one and and a fixed voltage on the other. The resistance and capacitance values are respectively given by:

$$R=\beta\frac{d}{A}$$

and

$$C=\eta d\,A$$

where β is the conductivity of the basalt, η is its thermal capacity, and *A* and *d* are respectively the area and thickness of the slab. The *A* is redundant really since unit area is assumed in the calculation.

The solar energy hitting the Moon's surface (solar total irradiance) is around 1600 W/m^2 . The lunar albedo according to the RASC Observer's Handbook is 0.12. So the maximum (from zenith) energy input rate to the Moon's surface is I_0 = 1600(1 - 0.12) = 1408 W/m^2 . The Moon rotates once with respect to the Sun once every 29.53 days. That is a "rotation" rate, Ω = 2.46 times 10^{-6} radians a second. So the heat input to our bit of flat, lunar crust is (assuming the Sun goes overhead):

$$I(t) = I_0 \sin(\Omega t)$$

when I is positive, and $I = 0$ otherwise (Sun below the horizon).

The rate of change of energy stored in the surface layer is:

$$dW_0 dt = I(t) - \sigma T_0^4 + (T_1 - T_0)/R \quad .$$

where σ is Stefan's constant, T_0 is the temperature of that surface layer and T_1 the temperature of the layer below. R is the resistance described above. The value of T_0 is given by

$$T_0 = W_0 C$$

W_0/C. For the layers below, the energy change formula ia a little different:

$$dW_i dt = (T_{i-1}^4 - T_i)/R + (T_{i+1} - T_i)/R$$

Then

$$T_i = W_i C$$

where $i = 0$ at the surface and increases downwards.

The equations are solved by initially setting all the layers to the same temperature, and then iterating down through the levels lots of times until the temperature gradient settles down. This is done here by doing the calculation every second. At this rate the input solar energy changes so slowly the iteration easily keeps up. We only write to the output file the values for intervals of 0.05 days, which all that is needed for output.

Radio Emission

The temperature calculation is well-defined by established measurements of thermal conductivity and capacity. The RF attenuation at 11.5 GHz is not. It can be found though by choosing a value that gets both the phase delay and the amplitude of the radio brightness temperature correct. The attenuation of the lunar soil is what we are really after here. So assume an attenuation coefficient α. Then the transmissivity ξ of one of the layers is

$$\xi = \exp(-\alpha d)$$

All layers are assumed to be the same. Then the noise temperature (N - we have already used T for something else) encountered on the upper surface of the ith layer is

$$N_i = \xi N_{i-1} + (1 - \xi) N_i$$

So we start at the bottom of the stack of rock layers and step to the surface. The noise temperature value at the surface is assumed to be what radiates to space.

Modelling Results

The model was run for 100 days in order for the impact of initial values to wear off. Here is a plot of the irradiance input and the temperature variations for every tenth of the one hundred layers, each 1 cm thick. The plot below shows the irradiance and the temperature variations for a single location on the lunar equator.

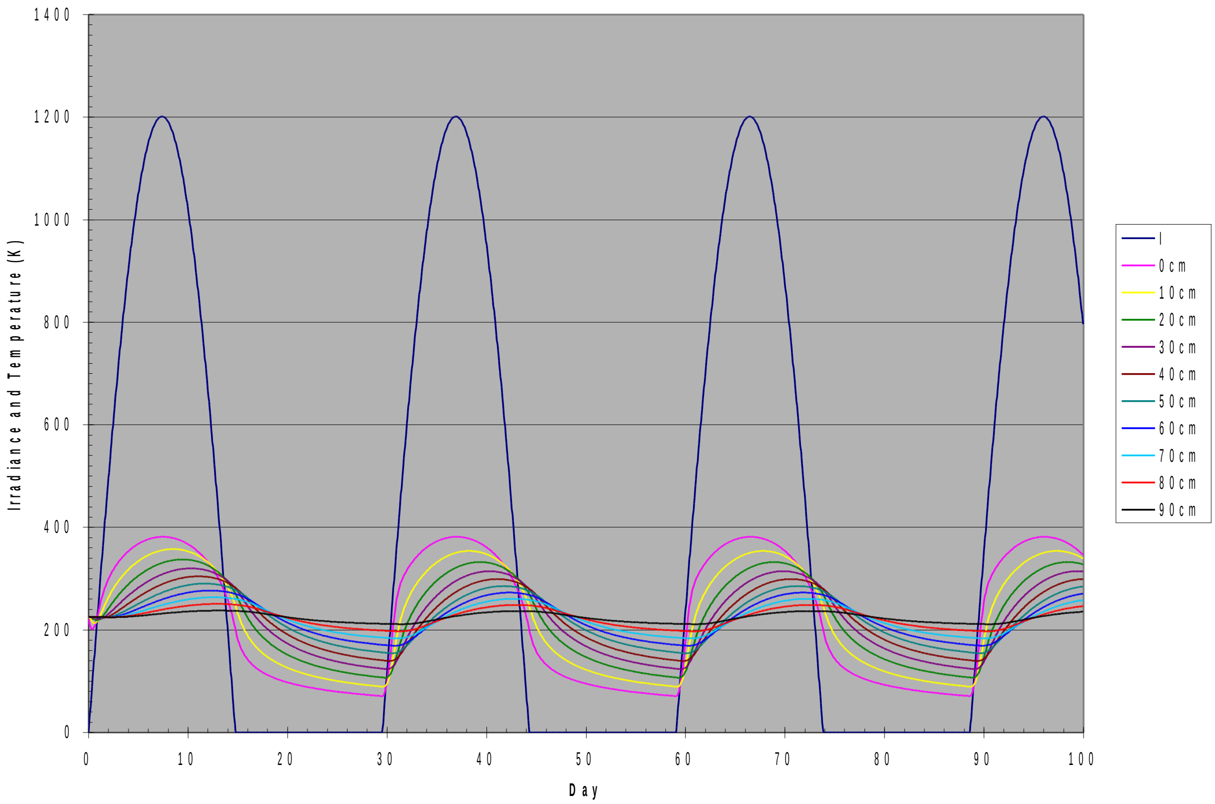

Figure 11: Heat input and modelled subsurface temperature changes in every tenth of the 100 layers as a function of lunar phase.

The figure below shows the plot with the high irradiance peaks taken out, so the temperature curves are more easily seen.

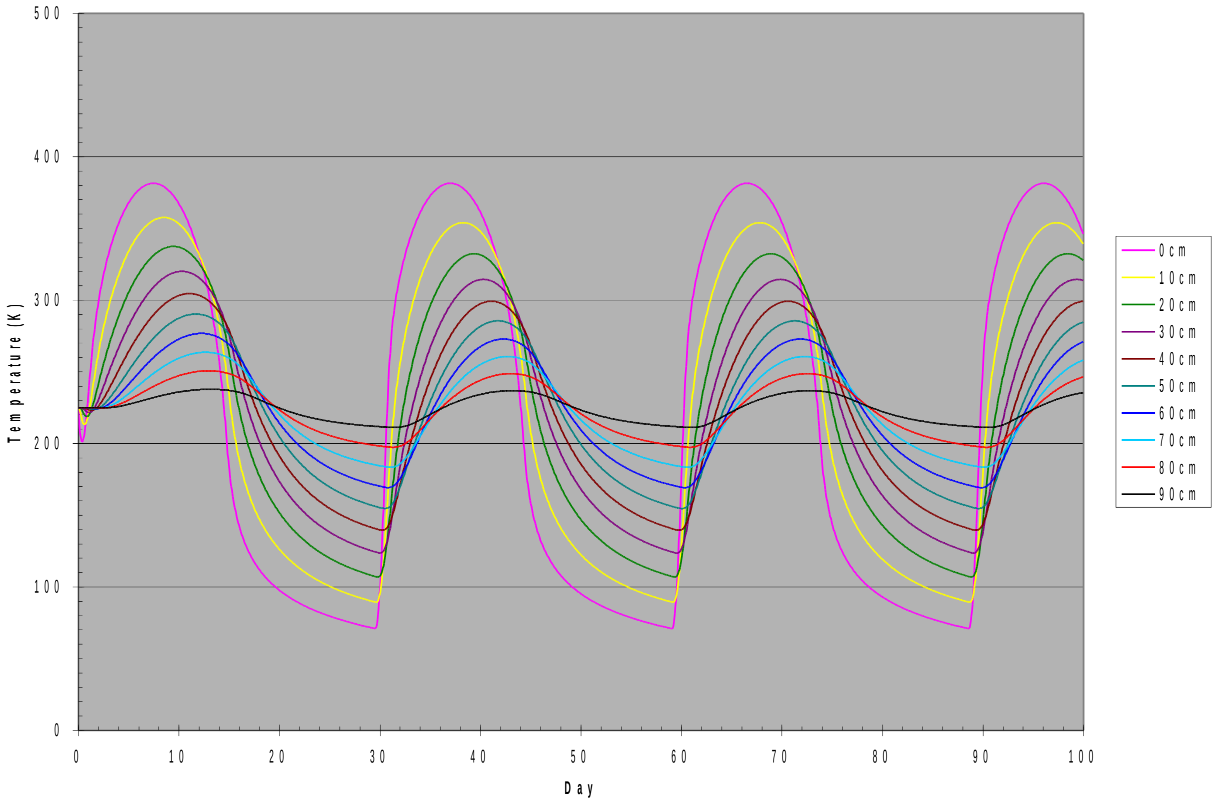

Figure 12: Temperatures as a function of depth and time. The phase lag with increasing depth is clearly visible.

The Radio Brightness Temperature

Disc Centre

The temperature at a depth of one metre is assumed to be the lunar mean temperature, taken here to be 225K. Then, we step our way to the surface one layer at a time:

$$Tb_{(j)} = aTb_{j-1} + (1-a)T_j$$

Where *j* identifies the layer, T_j the temperature of that layer as calculated by the heat flow model and *a* is the radio transmissivity at 2.6 cm wavelength. We assume that quantitity is constant throughout the rock/regolith column.

The attenuation that fits the delay was found to be 0.4m^{-1}, where the transmission coefficient is given by:

$$a = \exp(-0.4\mathrm{d})$$

where *d* is the thickness of the layer.
The figure shows the temperature variations at every 10cm depth, and the estimated radio brightness temperature values.

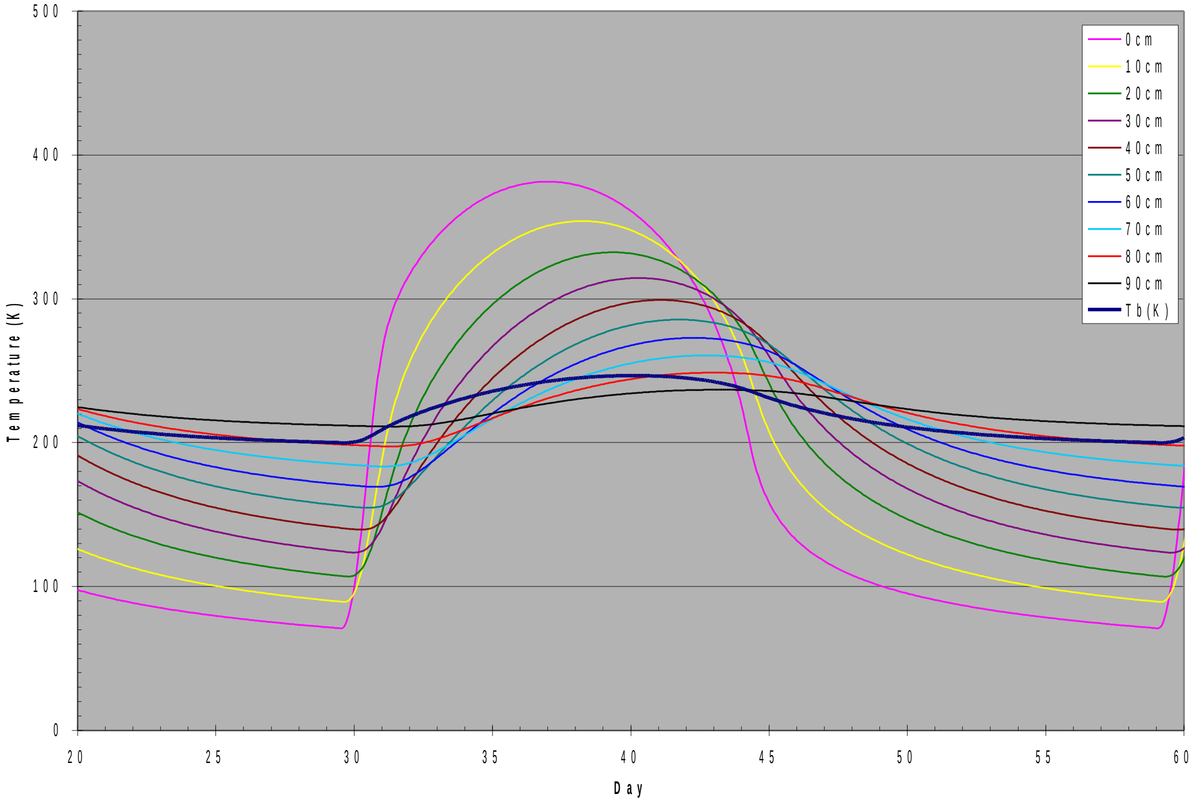

Figure 13: One cycle, showing the temperature variations as a function of depth and time, and the modelled radio brightness temperature (thick, black curve).There is a delay of about 2.5 days between the peak temperature at the surface and the peak radio brightness temperature at 2.6cm wavelength.

The Full Disc

When we look at the Full Moon, we see a uniformly illuminated disc. However, from a temperature point of view the situation is not uniform. On the west (right) side of the disc, it is sunset, and the surface is starting to cool off at the end of the lunar day. However, on the east (left) side of the disc, it is sunrise, and the surface is starting to warm up after a long, cold, lunar night. So where the subsoil is at in its thermal cycle depends on the position of the lunar disc. The radio telescope antenna is seeing all those points and measuring the integrated emission from all of them.

However, the 1.2 m dish "sees" the lunar limb with significantly less sensitivity than it sees the centre of the disc (about 10%), and taking into account other factors, such as foreshortening of each square metre of surface at the limb, the emission contribution from each point on the disc is weighted by a factor that is unity at the centre of the disc and significantly less at the limb.

In a very approximate way, ignoring high-latitude points on the lunar disc, the total emission, weighted according to distance from the centre of the disc, gives the result below.

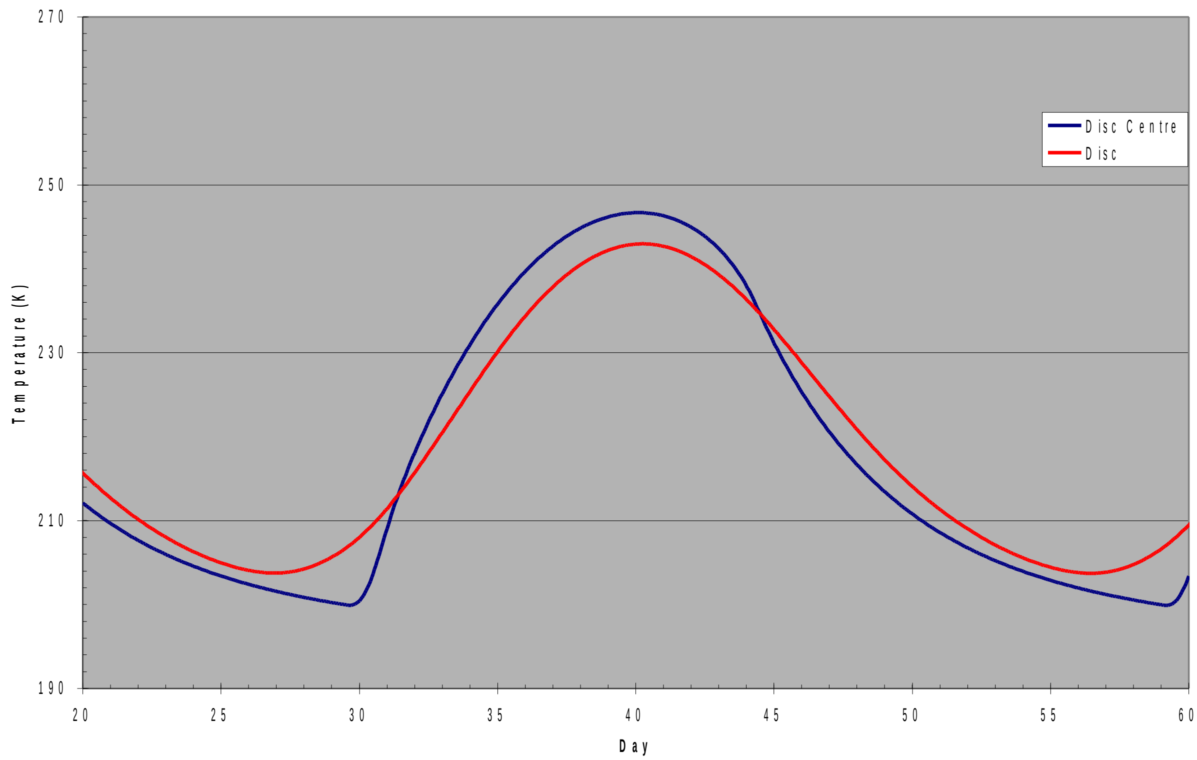

Figure 14: The modelled temperature variation for the centre of the disc (blue) is shown along with the modelled emission from the whole lunar disc (red), taking into account the antenna beam taper and foreshortening.

Comparison of Model with Observations

Each mean daily measurement is marked with a blue dot. The thick, red and blue lines shows the disc centre and "full disc" models respectively.

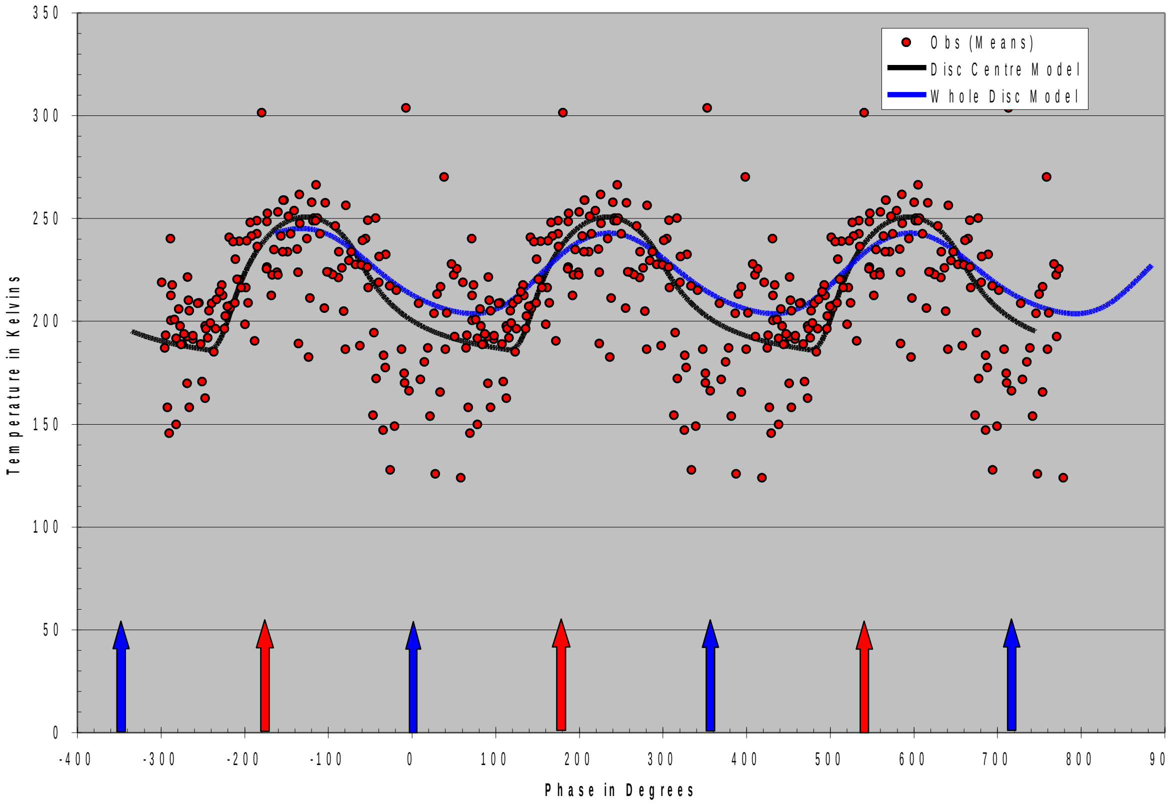

Figure 15: Data (three repeated cycles - red dots), the disc centre temperature model (black line) and an approximate full lunar disc model (blue line). New Moons are indicated by blue arrows and Full Moons by red arrows.

Comments

The radio brightess temperature at 2.6 cm wavelength lags the Full Moon by about 2.5 days, and varies between about 200 and 250 K, which is far less than the temperature variation. The scatter in the measurements was larger than hoped, but they were analyzeable. One clear, hot or cold days, the measurements were good. However, cloud and rain rapidly degraded the data. Then of course there were the usual problems in using single-antenna, total power systems. In hindsight this project could have been better done using an interferometer, although that would not have been possible with the existing hardware. It has been an enjoyable, challenging and educational experiment.

The next stage in this experiment is to make measurements at a shorter wavelength, 1.5 cm. Emissions at this wavelength originate closer to the surface and should show a smaller phase delay than the measurements at 2.6 cm. It is expected that tropospheric effects will be larger and rather than resorting once more to single-dish, total power measurements, as were used in the study described here, a custom-made small, correlation interferometer will be used. This should be more immune to tropospheric noise variations, and more challenging to use.

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